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Abstract

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Keywords: Temporal disaggregation, State space models, Structural time series models, Administrative data, Monthly GDP

JEL classification: E01, C32, C55, P44

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1 Introduction

The United Kingdom has traditionally relied on surveys to provide the data needed to compile the national accounts. Following the Bean Review (Bean, 2016), however, there has been an increased focus on the use of administrative data sources. A particular focus has been the use of Value Added Tax (VAT) returns to provide data on business turnover which are currently collected by the Monthly Business Survey (MBS). These data form the core of the output estimate of GDP, which has been published monthly since July 2018. This paper explores the role that VAT returns can play in delivering monthly estimates of business turnover.

The MBS sample is based on five strata. The first four strata are identified by employment size, as recorded in the Inter-Departmental Business Register. These are defined as employment groupings of 0-9, 10 to 49, 50 to 99 and 100 and over. Strata one to three are sampled, while stratum four is fully enumerated. The fifth stratum is comprised of firms with turnover in excess of £60 million even though their employment size would put them in bands one to three. This stratum is also fully enumerated.

All firms with a turnover greater than £83,000 are required to be registered for VAT, and they are all required to declare their turnover as well as any tax due. The number of firms registered for VAT is many times larger than the sample of the MBS, suggesting that these returns might be a better data source. There are, however, three problems to be overcome. First, there are issues of attribution for respondents with complicated business patterns; if a VAT return relates to turnover arising from a number of distinct business activities, the turnover needs to be allocated between those activities. A similar issue arises when more than one VAT returns relate to a single business unit. Apportionment is much more of a problem for firms in strata four and five, that is the largest businesses, than for those in strata one to three.

Secondly, VAT records accrue gradually. Information for December relating only to about ten percent of firms is available by the end of January, and the full set of returns covering December is not available until the end of April. Since there should be no coverage gain in using VAT returns for strata four and five, and there is both a time delay and an apportionment issue, it is likely that any use of VAT returns will be limited to strata one to three, while the MBS will remain in use for the largest businesses.

The most prominent issue to be addressed in the use of VAT data arises from the nature of the data. Approximately 10% of respondents make monthly returns, and there is also a small number of annual returns. Most respondents, however, make quarterly returns covering a period of three months, but within this, some of these cover the calendar quarter, some the three months ending in the first month of the calendar quarter and some the three months ending in the second month of the calendar quarter. In order to make use of these data it is necessary to generate monthly estimates from these aggregates. This paper addresses the issue of producing monthly VAT estimates from the full sample of returns, with the question of making best use of partial returns being left for subsequent work.

In this paper, we develop state space methods to disaggregate temporally noisy rolling quarterly VAT-based turnover. We estimate a new monthly measure of turnover for seventy-five industries and produce aggregate estimates covering approximately a quarter of gross value added in the UK economy. In addition to this extensive empirical work, our paper contributes to the literature on temporal disaggregation in two respects. First, we provide a discussion of the effect that noise in aggregate figures has on the estimation of disaggregated model components. Secondly, we adopt a new temporal aggregation strategy different from the method of Harvey and Pierse (1984). The method we use is easier to model and can be generalised to non-overlapping data.

In the next section we describe in detail the characteristics of the VAT data. In section 3 we discuss the literature on temporal disaggregation and the benefits of the state space approach. In section 4 we present our model and estimation strategy, which we illustrate in section 5 with one industry. We show the aggregate results for seventy-five industries in section 6 and conclude in the last section.

2 Description and characteristics of the data

We work from quarterly series provided by HMRC VAT returns that show three-month turnover totals from seventy-five industries. The list of industries with their SIC codes and descriptions can be found in appendix A. We work only from data for firms in size bands one to three, that is to say small and medium size businesses, and we observe the raw data which are not seasonally adjusted. The time series include seventy data points, from March 2011 to December 2016. Our aim is to disaggregate temporally these quarterly data into monthly figures, and three characteristics of the data complicate this task.

First, as explained above quarterly VAT data are available on a monthly frequency. Firms reporting quarterly can start reporting in any month, generating three possible quarterly reporting patterns, which we refer to as quarterly *staggers*. There is also one monthly stagger and twelve annual staggers made of firms reporting monthly and annual totals respectively, but the quarterly data are weighted to represent all of those; hence we observe rolling three-month quarterly turnover figures representing all band one to three firms reporting VAT. Table 1 gives an illustration of the quarterly staggers. Temporal disaggregation methods rest on the minimisation of a given function of the data subject to the temporal aggregation constraints. In our case we constrain the three-month sums of the monthly figures we generate to be equal to the quarterly totals that we observe. The rolling nature of the data means that each month is constrained by three quarterly totals, leaving very few degrees of freedom.

Table 1: Representation of the quarterly VAT-based turnover data reporting patterns

	Month												
	J	F	M	A	M	J	J	A	S	O	N	D	J
Stagger 1			x			x			x			x	
Stagger 2	x			x			x			x			x
Stagger 3		x			x			x			x		

x = quarterly turnover total

An unattractive feature of the staggers is that they are unequally populated. The calendar quarter in particular is the most populated in general. This bias across staggers should be alleviated when the quarterly data are weighted to represent the entire VAT reporting firms. However, the weighting procedure might not be entirely satisfactory if the bias is rapidly changing over time. This is something that we account for in our model.

The most challenging feature of the data arises from the amount of noise they exhibit. Constraining the monthly figures to be coherent with noisy quarterly totals generates estimates that are highly erratic and not sensible. As a result, we need to relax the rolling quarterly constraints and account for potential observation errors, which implies estimating the noise in the data. This appears to be challenging because separating the natural volatility embedded in the series from the noise they are subject to is difficult with a relatively short sample.

We show the aggregate data of the three series of interest in Figure 1. These are the weighted quarterly data for firms in size bands one to three, the MBS data for the largest businesses and the MBS for firms in size bands one to three. We use the latter as a comparison to the VAT monthly estimates and as a 'synthetic' series to test our methods. To derive the aggregate series we weighted the industries' series with two weight factors. We weight the size bands series according to the contribution of the particular size bands to the overall industry turnover[1] and the contribution of the industry to the overall gross value added in the economy. This weighting is necessary because, while the output measure of GDP is produced from gross output indicators, movements in these are combined using net output weights. In particular, a simple aggregation would overstate the importance of the distribution sector whose gross output is measured by margin and not turnover. To facilitate comparison the aggregate series are indexed to September 2011 = 100.

From this chart it is hard to get a good sense of the underlying series; there are clear seasonal movements. In order to suppress the effects of these, we show in Figure 2 moving averages of the three series over the period September 2011 - June 2016. Each is calculated over a thirteen-month period with the two outer-most months given half-weights. The series are centred on the seventh month of the moving average window. In order to facilitate comparison the three series are indexed to September 2011 = 100. The chart strongly suggests that the underlying movement in the VAT series is materially different from that shown by MBS for bands one to three, even though the two series relate to the same types of businesses and, over our sample period as a whole, show similar growth. Taking this divergence as given, we now present how to extract the monthly path in the rolling quarterly VAT-based turnover figures.

[1] Firms in size bands one to three account for approximately 39% of turnover on average in our sample.

Figure 1: Levels of the aggregated raw series, index September 2011 = 100

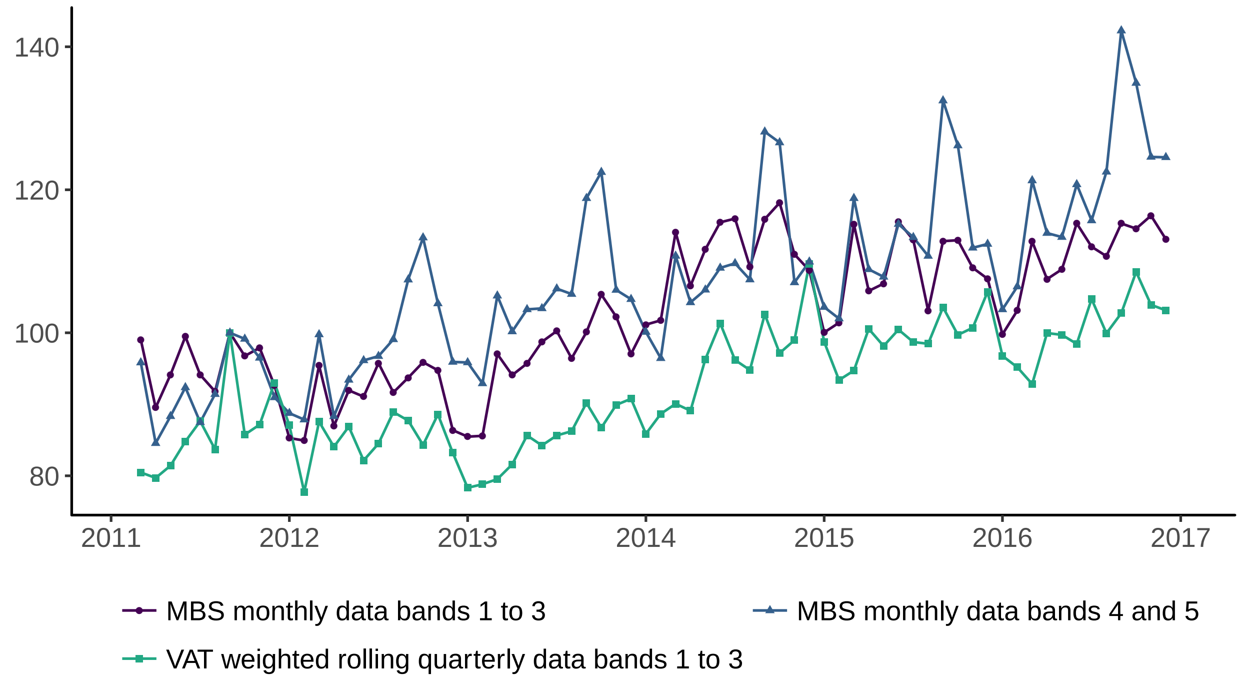
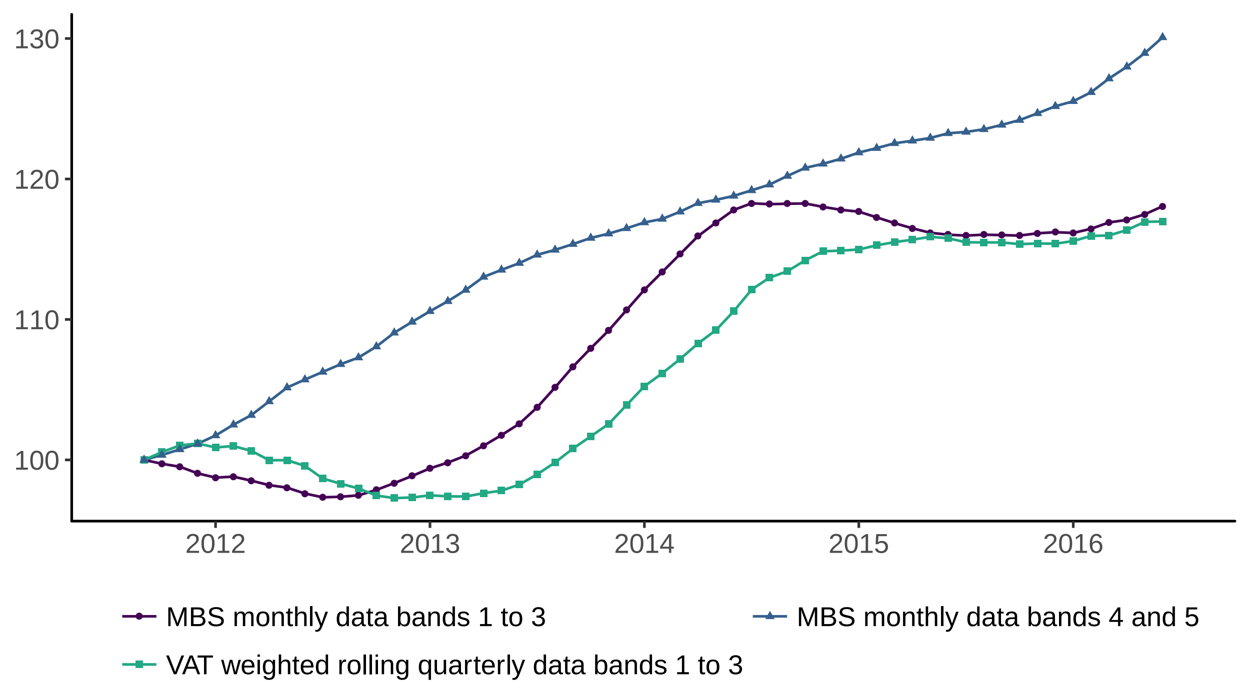


Figure 2: Thirteen-month moving averages of the aggregated series, index September 2011 = 100



3 State space methods as temporal disaggregation approach

There are a number of ways in which the issue of temporal disaggregation of the VAT returns can be addressed. All methods rely on constraining the interpolands, which are the high frequency estimates, to the low frequency totals that are observed. When potentially related series are available at the desired high frequency, those can be used in the estimation to identify the path in the interpolands. These series are usually referred to as indicator, related or covariate series.

Most commonly used methods, which are extensively exploited in national statistical institutes, are least-squares techniques. They rely on the constrained minimisation of a given residual, where the observed aggregate totals serve as aggregation constraints. These techniques include regression-based methods such as Chow and Lin (1971), Fernandez (1981) and Litterman (1983) to make use of indicator series. If no related series are available at the desired frequency, the preferred method is Boot, Feibes and Lisman (1967), which is equivalent to Fernandez's approach with a constant indicator series. Mitchell, Smith, Weale, Wright and Salazar (2005) generalise least-squares techniques in a single framework and derive an approximation to use data in logarithms. However, these methods do not yield satisfactory results when applied to the particular problem we face.

Several characteristics of the VAT data such as their dynamic seasonality, rolling nature and the noise they exhibit render their temporal disaggregation problematic. Applying least-squares techniques to the rolling quarterly figures produces highly erratic estimates. This stems from the difficulty in estimating observation errors and the impossibility of capturing stochastic model components. Consequently, we choose to develop state space methods, which do not suffer from either of these shortcomings. We are able to filter and disaggregate temporally the quarterly VAT figures in a single framework.

The Kalman filter is an efficient algorithm for estimating time-varying components such as trends, slopes and seasonal effects, as well as potential observation errors, when those are modelled together in a state space form. State space models have been applied to temporal disaggregation problems since the seminal paper of Harvey and Pierse (1984), and an exhaustive literature has emerged on this topic. Notably, Moauro and Savio (2005) extend this approach further by specifying a multivariate model to take advantage of related series. In our case we can use the MBS for the largest businesses as an indicator series since it will remain the measure of turnover for these businesses. Proietti and Moauro (2006) show how to deal with nonlinearities arising from the temporal aggregation constraints when the data are in logarithms.

These studies, however, provide little discussion of the effect that noise in aggregate figures has on the estimation of disaggregated model components, and they do not analyse overlapping data. We now present a state space model designed to accommodate these particular features of the data.

4 A nonlinear structural time series model

The characteristics of the VAT data lead us to adopt a state space framework as our preferred estimation strategy. State space methods provide flexible and efficient techniques for filtering and temporally disaggregating the data. In particular, we develop a nonlinear multivariate structural time series model. In this section, we present the different components of the model, cast it in state space form, and describe our estimation strategy.

We follow the Unobserved Components literature and model the seasonally adjusted monthly turnover figures with a local linear trend model. To capture the seasonality and stagger bias we use a single seasonal dummy model. Calendar variations can also affect business turnover in some industries and we discuss possible methods of accounting for those. We then describe the aggregation strategy we use to relate the monthly model underlying the interpolands and the quarterly totals that we observe. We notably use a nonlinear aggregation function to accommodate data in logarithms; this mitigates the risk of heteroscedasticity while recognising that the aggregations constraints are linear. Considering that the MBS for the largest businesses could provide valuable information on the monthly path in the interpolands, we extend the model to a multivariate structure.

We then cast the model in a state space form. Since the model is nonlinear we present the method we use to derive an approximating linear model, where the approximation can be made as precise as required. The latter can then be estimated using standard state space methods based on the kalman filter and smoother. Finally, we show how to detect and treat outlying observations in the covariate series to improve the estimation.

4.1 The local linear trend model

Economic time series can often be decomposed into distinct components, which can be stochastic or deterministic. For variables closely related to the economic activity, these usually include a trend, a seasonal component and an irregular component and/or an observation error. We choose to adopt the Unobserved Components approach, as set out in Harvey (1989) and Durbin and Koopman (2012). Specifically, we assume that the seasonally adjusted interpolands follow the local linear trend model

$$\begin{aligned} x_t &= \mu_t + e_t, & & \\ \mu_{t+1} &= \mu_t + \nu_t + \xi_t, & \xi_t &\sim \mathrm{N}(0, \sigma_\xi^2), \\ \nu_{t+1} &= \nu_t + \zeta_t, & \zeta_t &\sim \mathrm{N}(0, \sigma_\zeta^2), \end{aligned} \tag{1}$$

where x_{1t} is the logs of the seasonally adjusted interpolands, μ_t a stochastic trend, and e_t the irregular variation. The disturbance terms ξ_t and ζ_t are assumed to be serially and mutually uncorrelated. The interpolands are thus composed of a time-varying trend but can also be subject to irregular variations in their level. We let the slope in the trend slowly vary over time by modelling it as a random walk. Turnover series at industries level can exhibit rapidly changing trends, which this model is thus suited to capture efficiently.

A simple approach to model the irregular variations e_t is to assume a white noise model. However, the irregular component can sometimes exhibit serial correlation. To account for this possibility we model the irregular variations as a general autoregressive model of order p, such that

$$\Phi(B)e_{t+1} = \kappa_t, \qquad \kappa_t \sim \mathrm{N}(0, \sigma_\kappa^2) \tag{2}$$

where $\Phi(B) = (1 - \phi_1 B - \phi_2 B^2 - ... - \phi_p B^p)$ and B is the backshift operator such that $Be_t = e_{t-1}$. We consider $p = 0, 1, 2$.

4.2 Seasonal components

We model proportionate seasonal effects as stochastic dummy variables, such that

$$\gamma_{t+1} = -\sum_{j=1}^{11} \gamma_{t+1-j} + \omega_t, \qquad \omega_t \sim N(0, \sigma_\omega^2), \tag{3}$$

where the γ_t represent the three-month seasonal effects. Since we include a stochastic term ω_t, the twelve-month sums of the seasonal effects do not exactly add up to zero, but they do on average over the estimation period.

It is not possible to extract monthly seasonal effects from the observation of the three-month aggregate figures. This is because, if we assume that the seasonal pattern repeats itself over the years, two of the twelve three-month seasonal effects are linearly dependent of the other ten. This means that the linear system expressing the aggregate seasonal effects as a function of the monthly seasonal effects is under-identified. Nevertheless, not being able to extract monthly seasonal effects instead of three-month seasonal effects is not an obstacle to the estimation of monthly seasonally adjusted figures.

An alternative approach to model the seasonality is to use a trigonometric form. Proietti (2000) shows that the trigonometric form, under certain restrictions, is equivalent to the quasi random walk model of Harrison and Stevens (1976), and that it usually outperforms the dummy seasonal model. This is because the quasi random walk model is better suited to seasonal movements that vary slowly over time, which is most often the case. However, two characteristics of the VAT data suggest that the dummy variable specification might be more appropriate for our model.

First, the VAT figures can be subject to a time-varying stagger bias, which adds to the typical seasonality and cannot be separated from it. This stems from the impossibility of identifying monthly seasonal effects. In addition, we expect the stagger bias to change more rapidly over time than typical seasonal movements. This means that the volatility embedded in the seasonal effects that are captured in our model is particularly high.

Secondly, the noise in the data blends in the disturbances of the seasonal effects. We could try to capture this noise by specifying an observation error, but we would face an identification issue. This is because both the seasonal disturbances and the observation errors are represented by white noise and are modelled at the aggregate level.

4.3 Calendar variations

Seasonal models are designed to capture movements in the data that occur on a regular frequency and whose magnitude changes relatively slowly over time. However, some seasonal effects are subjected to frequent and regular changes in their periodicity, and this can generate biases in the estimated seasonality. These effects are usually referred to as calendar effects and can be classified in two broad categories: moving festivals and trading days effects.

The Easter period is the only significant moving festival in the UK. Easter can fall either in March or in April and can overlap both months. The common approach to estimate Easter effects is due to Bell and Hillmer (1983) and consists of setting

$$E_t = \beta h_t,$$

with E_t the Easter effect at period t, and h_t the proportion of the total number of days H_t over the Easter period that falls in month t. However, the Easter effects modelled as such do not add up to zero over a year. One way to remedy this issue is to adopt a representation similar to Harvey (2006) and set[2]

$$E_t = \beta(h_t - \sum_{t=1}^{s} h_t/s) = \beta h_t^a \tag{4}$$

with s the frequency, which is twelve in our case, and $\sum_{t=1}^{s} h_t = 3$. The three-month aggregates ending in April and May include both March and April; hence two months for which h_t is equal to one.

In the US, businesses can see their turnover fluctuate considerably round the seven days preceding Easter and it is usual to set $H_t = 7$. However, Russ and Aziz (2017) suggest using a specific Easter period for the UK. Their results show that it is better to account for the period Good Friday to Easter Monday, thus accounting for the entire bank holiday period, or from Monday before Easter Sunday to Friday following it. We choose the former and set $H_t = 4$ as the number of days that precede Easter Monday.

The trading days effects are usually insignificant with quarterly data, as the varying number of trading days in a month partially averages out over three months (X-13ARIMA-SEATS reference manual, US Census Bureau[3]).

4.4 Temporal aggregation

The additive unobserved component decomposition is usually more suitable for data in logarithms. This is because modelling the decomposition in logs allows us to account for multiplicative model specifications, such as proportional seasonal and Easter effects, and potential heteroskedasticity in the irregular variations. However, the temporal aggregation constraints do not hold any more when modelled in logarithms because the logarithm of the sum is not equal to sums of the logarithms. To get round this issue we need to model the aggregate

[2] Proietti and Moauro (2008) use a different strategy and subtract the long term average of h_t, such that $E_t = \alpha(h_t - \bar{h_t})$ with Easter falling on average 35.4% of the time in March and 64.6% of the time in April over the first four hundred years of the Gregorian calendar. This method is directly applicable to rolling quarterly data by setting $\bar{h_t} = 0.354$ in March and $\bar{h_t} = 0.646$ in June, while $\bar{h_t} = h_t = 1$ in April and May and zero otherwise. This is also the method implemented in X-13ARIMA-SEATS and X-12-ARIMA (see Findley, Monsell, Bell, Otto and Chen (1998)).

[3] https://www.census.gov/ts/x13as/docX13ASHTML.pdf

VAT figures as a nonlinear function of the state components. We model the aggregate VAT figures that we observe as

$$y_t = \log(e^{x_t} + e^{x_{t-1}} + e^{x_{t-2}}) + \gamma_t + \beta_t h_t^a, \quad t = 1, ..., N \tag{5}$$

where y_t are the three-month VAT turnover aggregates, x_t are the seasonally adjusted monthly interpolands and γ_t are the three-month seasonal effects. The observation equations (5) serve as the aggregation constraints. Even though the data are subject to reporting errors, we choose not to include an observation error in the observation equation. As we mentioned while setting out the seasonal model, we cannot separate the observation errors from the seasonal disturbances, and we chose to let the seasonal disturbances capture the noise in the data.

The temporal aggregation technique we use differs from the widely used method introduced by Harvey and Pierse (1984) and does not rely on the augmentation of the state vector with a cumulator variable. Our approach suits the rolling nature of the data better, and avoiding the augmentation of the state vector offers several benefits. Among those is the direct observability of the interpoland estimates and their standard errors. Augmenting the state vector with a cumulator variable requires a decumulation step post estimation, while a second augmentation is required to observe the interpolands' standard errors. Another advantage of our method is its simplicity, whereas the state augmentation method can be difficult to grasp at first and tricky to model.

Importantly, the overlapping aggregation function is generalisable to more common temporal disaggregation problems. For instance, it is more usual to observe quarterly data on a quarterly frequency and not on a monthly frequency. In this case we can still approach the series we observe as a rolling series, but with the two intermediate values in-between calendar quarters missing. Missing values are then handled straightforwardly by the Kalman filter.

4.5 Using an indicator series

Temporal disaggregation techniques often rely on indicator series available on the same frequency as the interpolands that can help identify the variations in the latter. These techniques make use of the correlation between the observed aggregates subject to interpolation and the indicator series at a lower frequency. Hence, they rely on the assumption that the correlation between the low frequency aggregates applies to the high frequency estimates.

Most commonly used techniques for temporal disaggregation with mean of indicator series are regression based methods and include Chow and Lin (1971), Fernandez (1981) and Litterman (1983). An important issue with those methods is that, with data that are not seasonally adjusted, the estimation procedure must be carried out in two steps. We use the indicator series to improve the estimation of the nonseasonal variations in the interpolands, but with the regression methods we are not able to estimate the seasonal effects in both the interpolated and related series simultaneously. Hence, regression methods require the indicator series to be seasonally adjusted in a first step, while the temporal disaggregation happens in a second stage. Separately, regression methods imply a causal relationship between the covariate series and the interpolated figures. These problems can be alleviated by using Seemingly Unrelated Time Series Equations (SUTSE) state space models.

SUTSE models are structural time series models that have been developed by Harvey (1989) and Harvey and Koopman (1997). Moauro and Savio (2005) notably apply SUTSE models to temporal disaggregation problems. Unlike with regression methods, we can make use of an indicator series whilst making relatively weak assumptions on the form that the relationship between the interpolands and the covariates can take – we simply assume that both series are affected by a common environment. In addition, we can let both series follow distinct trends and use covariates that are not seasonally adjusted by estimating seasonal effects for both the covariates and the interpolands simultaneously. Finally, the regression methods can be implemented in a SUTSE framework; thus there is no loss of generality by adopting this approach.

The covariate series we use are the monthly MBS estimates for bands four and five, which we model as the sum of the seasonally adjusted figures, the seasonal effects, and the Easter effect given by (1), (3), and (4) respectively. Modelling the target and covariate series in a multivariate framework thus leads to the following model

$$\begin{aligned}
y_{1,t} &= \log(e^{x_{1,t}} + e^{x_{1,t-1}} + e^{x_{1,t-2}}) + \gamma_{1,t} + \beta_{1,t} h^a_{1,t}, \\
y_{2,t} &= x_{2,t} + \gamma_{2,t} + \beta_{2,t} h^a_{2,t}, \\
\boldsymbol{x}_t &= \boldsymbol{\mu}_t + \boldsymbol{e}_t, \\
\Phi(L)\boldsymbol{e}_{t+1} &= \boldsymbol{\kappa}_t, && \boldsymbol{\kappa}_t \sim \mathrm{N}(0, \Sigma_\kappa), \\
\boldsymbol{\mu}_{t+1} &= \boldsymbol{\mu}_t + \boldsymbol{\nu}_t + \boldsymbol{\xi}_t, && \boldsymbol{\xi}_t \sim \mathrm{N}(0, \Sigma_\xi), \\
\boldsymbol{\nu}_{t+1} &= \boldsymbol{\nu}_t + \boldsymbol{\zeta}_t, && \boldsymbol{\zeta}_t \sim \mathrm{N}(0, \Sigma_\zeta), \\
\boldsymbol{\gamma}_{t+1} &= -\sum_{j=1}^{11} \boldsymbol{\gamma}_{t-j} + \boldsymbol{\omega}_t, && \boldsymbol{\omega}_t \sim N(0, \Sigma_\omega), \\
\boldsymbol{\beta}_{t+1} &= \boldsymbol{\beta}_t.
\end{aligned} \tag{6}$$

where $y_{1,t}$ are the aggregate VAT figures and $y_{2,t}$ are the covariates. The state components $\boldsymbol{\mu}_t$, $\boldsymbol{\nu}_t$, $\boldsymbol{\gamma}_t$, $\boldsymbol{e}_t$ and $\boldsymbol{\beta}_t$ and the state disturbances $\boldsymbol{\xi}_t$, $\boldsymbol{\omega}_t$, $\boldsymbol{\zeta}_t$ and $\boldsymbol{\kappa}_t$ are vectors of length two with the interpoland's component as first term and the covariate's component as second term.

We model deterministic seasonal effects for the covariate series; that is $\omega_{2,t}$ is null. This comes from the difficulty, with a small sample, of separating the irregular variations in the seasonally adjusted covariates from the disturbances in the seasonal component. It is similar to the problem occurring when trying to distinguish the VAT seasonal disturbances from the aggregate noise.

We assume that the disturbances are uncorrelated across time and across unobserved components, but there can be a contemporaneous correlation across the VAT and MBS series. We define the covariance matrix Σ_h, with $h = \kappa, \xi, \zeta$, as

$$\Sigma_h = \begin{pmatrix} \sigma^2_{1,h} & \rho_h \sigma_{1,h} \sigma_{2,h} \\ \rho_h \sigma_{1,h} \sigma_{2,h} & \sigma^2_{2,h} \end{pmatrix}$$

with $\sigma^2_{1,h}$ the variance of the interpoland's h component and the $\sigma^2_{2,h}$ the variance of the covariate's h component.

In general, state space models yield trend and slope estimates that are too smooth to exhibit white noise disturbances, as we assume in our model. Notably, Harvey and Koopman (1992) show that, although the state disturbances are assumed to be serially uncorrelated, their estimators often exhibit high serial correlation. This should not be ignored when estimating a multivariate model. High serial correlation affects the estimated correlation coefficients and can yield to spurious estimates.

As far as we are aware, this is not an issue that is raised in the literature on multivariate state space models. In their SUTSE estimation, Moauro and Savio (2005) find correlation coefficients equal to unity, which is not surprising given the serial correlation that state disturbances can exhibit. We are inclined to see these high absolute correlation coefficients as spurious. For this reason, we decide to let the irregular disturbances in the seasonally adjusted monthly estimates be correlated, while the correlation coefficients in the trend and slope disturbances are fixed to zero. This amounts to estimating the trend and slope components in the interpolated series independently from the covariate series.

These restrictions should not have important implications. First, the interpolated and covariate series do not represent the same businesses : while the VAT data are constructed from small and medium size businesses, the MBS series we use as covariates represent the largest businesses only. It is common to observe different trends in the business activity of these two types of businesses. Secondly, the rolling nature of the data means that we have enough information to estimate the trend and slope components in the interpolands efficiently, and very little can be gained by letting these components be correlated across series. It is only regarding the irregular monthly variations in the seasonally adjusted estimates that we believe the MBS can be helpful. [4]

4.6 State space representation and estimation

A state space representation of model (6) is

$$\begin{aligned} \boldsymbol{y}_t &= Z_t(\boldsymbol{\alpha}_t), \\ \boldsymbol{\alpha}_{t+1} &= T\boldsymbol{\alpha}_t + R\boldsymbol{\eta}_t, \qquad \boldsymbol{\eta}_t \sim \mathrm{N}(0, Q), \\ \boldsymbol{\alpha}_1 &\sim \mathrm{N}(\boldsymbol{a}_1, P_1), \end{aligned} \tag{7}$$

with $\boldsymbol{\alpha}_t = (\boldsymbol{\alpha}'_{1,t}, \boldsymbol{\alpha}'_{2,t})'$; $\boldsymbol{\alpha}_{1,t} = (\mu_{1,t}, \mu_{1,t-1}, \mu_{1,t-2}, \nu_{1,t}, \gamma_{1,t}, ..., \gamma_{1,t-10}, e_{1,t}, e_{1,t-1}, e_{1,t-2}, \beta_{1,t})'$, $\boldsymbol{\alpha}_{2,t} = (\mu_{2,t}, \nu_{2,t}, \gamma_{2,t}, ..., \gamma_{2,t-10}, e_{2,t}, e_{2,t-1}, \beta_{2,t})'$, and where the system matrices are defined as

$$\begin{aligned} Z_t &= \mathrm{diag}(Z_{1,t}, Z_{2,t}), \\ T &= \mathrm{diag}(T_{1,LLT}, T_{1,\omega}, T_{1,e}, T_{1,\beta}, T_{2,LLT}, T_{2,\omega}, T_{2,e}, T_{2\beta}), \\ R &= \mathrm{diag}(R_{1,\mu}, R_{1,\nu}, R_{1,\omega}, R_{1,e}, R_{1,\beta}, R_{2,\mu}, R_{2,\nu}, R_{2,\omega}, R_{2,e}, R_{2,\beta}), \\ Q &= \mathrm{diag}(\sigma^2_{1,\xi}, \sigma^2_{1,\zeta}, \sigma^2_{1,\omega}, \sigma^2_{1,\kappa}, 0, \sigma^2_{2,\xi}, \sigma^2_{2,\zeta}, \sigma^2_{2,\omega}, \sigma^2_{2,\kappa}, 0) \quad \text{and} \quad Q[4,9] = Q[9,4] = \rho_\kappa \sigma_{1,\kappa}\sigma_{2,\kappa}. \end{aligned}$$

[4] We have investigated the implications of restricting the correlation in the trend and the slope components with a set Monte Carlo simulations. The results show that this has a negligible effect on the estimation.

with $Z_{1,t} = (1, 1, 1, 0, 1, 0, ..., 0, 1, 1, 1, h^a_{1,t})$; $Z_{2,t} = (1, 0, 1, 0, ..., 0, 1, 0, h^a_{2,t})$;

$$T_{1,LLT} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};\ T_{2,LLT} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix};\ T_{1,e} = \begin{pmatrix} \phi_{11} & \phi_{12} & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix};\ T_{2,e} = \begin{pmatrix} \phi_{21} & \phi_{22} \\ 1 & 0 \end{pmatrix};$$

$$T_{1,\omega} = T_{2,\omega} = \begin{pmatrix} -1 & -1 & ... & -1 & -1 \\ 1 & 0 & ... & 0 & 0 \\ 0 & 1 & ... & 0 & 0 \\ . & . & ... & . & . \\ 0 & 0 & ... & 1 & 0 \end{pmatrix};\ T_{1,\beta} = T_{2,\beta} = 1;\ R_{1,\mu} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}';\ R_{2,\mu} = R_{1,\nu} =$$

$R_{2,\nu} = 1$; $R_{1,\omega} = R_{2,\omega} = \begin{pmatrix} 1 & 0 & ... & 0 & 0 \end{pmatrix}'$; $R_{1,e} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}'$;$R_{2,e} = \begin{pmatrix} 1 & 0 \end{pmatrix}'$; $R_{1,\beta} = R_{2,\beta} = 0$. We express Q using the Cholesky factorisation as $Q = LL'$ and minimise the negative log-likelihood function with respect to the parameters in the lower triangular matrix L. Thus, we ensure that the estimated state disturbance matrix Q is positive semi-definite.

Table 2 shows the dimensions of the state space objects. We observe the three-month VAT turnover aggregates from the third period onward; hence the first two terms of y_t are missing values.

Table 2: Dimensions of the state space objects of model (7)

Object	Dimension	Object	Dimension
$\boldsymbol{y}_t$	2×1	$\dot{Z}_t$	2×35
$\boldsymbol{\alpha}_t$	35×1	T	35×35
$\boldsymbol{\eta}_t$	35×1	R	35×10
		Q	10×10

The Kalman filter and smoother cannot be applied directly to model (7) because the observation function $Z_t(.)$ is a nonlinear function of the states. The approximation for nonlinear aggregation constraints of Mitchell et al. (2005) is not a viable option in our case because it rests on the assumption that the monthly changes in the data are relatively small, whereas the VAT data are subject to strong seasonal movements. Consequently, we must resort to nonlinear estimation methods.

Linearisation method

The best strategy when the data are subject to strong seasonal movements is to follow Proietti and Moauro (2006), who apply a *sequential linear constrained* (SLC) method to estimate a model with nonlinear aggregation constraints. This algorithm is explained further in Proietti (2006) in the general context of nonlinearly aggregated mixed models. When the model is Gaussian, as in our case, this approach is equivalent to the linearisation by mode estimation as set out in Durbin and Koopman (2012).

An alternative approach could be to use the Extended Kalman filter. This method consists of running the Kalman filter on the nonlinear model, where at every step of the filter the observation equation is linearised with a first order Taylor approximation at the one step ahead prediction that is yielded by the previous step, with the initial value of the state vector as an approximation point for the first step. This approach has two significant disadvantages compared to the SLC method. First, it cannot be readily employed with statistical packages because few offer an Extended Kalman filter routine. Secondly, and more importantly, it would yield discrepancies in the aggregation constraints because the approximation errors cannot be reduced to a chosen tolerance value. These discrepancies can be significant if the month-on-month changes in the interpolands are large.

The SLC algorithm is an iterative method which consists in estimating a model with an approximated linear constraint, where the approximation is sequentially improved using the solution of the estimation as a new guess until convergence. The approximation error can thus be reduced to zero. This is a critical aspect for us because national statistical institutes attach a lot of importance to respecting aggregation constraints exactly. We linearise the observation equation by taking the first order Taylor expansion of $\mathrm{Z}_t(.)$ at a guess $\tilde{\boldsymbol{\alpha}}_t$, which yields

$$\boldsymbol{y}_t = \mathrm{Z}_t(\tilde{\boldsymbol{\alpha}}_t) + \dot{\mathrm{Z}}_t.(\boldsymbol{\alpha}_t - \tilde{\boldsymbol{\alpha}}_t), \qquad \text{where} \quad \dot{Z}_t = \left.\frac{\partial \mathrm{Z}_t(x)}{\partial x}\right|_{x=\tilde{\boldsymbol{\alpha}}_t}.$$

Since $\mathrm{Z}_t(\tilde{\boldsymbol{\alpha}}_t)$ and $\dot{\mathrm{Z}}_t.\tilde{\boldsymbol{\alpha}}_t$ are not random quantities, but instead are fixed prior to the estimation, we can define a new observation vector $\tilde{\boldsymbol{y}}_t$ such that

$$\tilde{\boldsymbol{y}}_t = \dot{\mathrm{Z}}_t.\boldsymbol{\alpha}_t, \qquad \text{where} \quad \tilde{\boldsymbol{y}}_t = \boldsymbol{y}_t - \mathrm{Z}_t(\tilde{\boldsymbol{\alpha}}_t) + \dot{\mathrm{Z}}_t.\tilde{\boldsymbol{\alpha}}_t. \tag{8}$$

By replacing the observation equation in (7) with the approximated linear observation equation (8) we get the approximating linear model

$$\begin{aligned} \tilde{\boldsymbol{y}}_t &= \dot{\mathrm{Z}}_t.\boldsymbol{\alpha}_t, \\ \boldsymbol{\alpha}_{t+1} &= T\boldsymbol{\alpha}_t + R\boldsymbol{\eta}_t, \qquad \boldsymbol{\eta}_t \sim \mathrm{N}(0, Q), \\ \boldsymbol{\alpha}_1 &\sim \mathrm{N}(\boldsymbol{a}_1, P_1), \end{aligned} \tag{9}$$

which can be estimated using the standard Kalman filter and smoother. We can now set out the SLC algorithm which, following Proietti and Moauro (2006) is

Step 1: Generate model (9) at a guess $\tilde{\boldsymbol{\alpha}}_t$;

Step 2: Run the Kalman filter and smoother on model (9), which yields the smoothed state vector $\hat{\boldsymbol{\alpha}}_t$;

Step 3: Set $\tilde{\boldsymbol{\alpha}}_t$ to $\hat{\boldsymbol{\alpha}}_t$;

Step 4: Iterate steps 1 to 3 until $\hat{\boldsymbol{\alpha}}_t = \tilde{\boldsymbol{\alpha}}_t$.

Once the algorithm has converged we can observe that

$$\begin{aligned} \boldsymbol{y}_t &= \mathrm{Z}_t(\tilde{\boldsymbol{\alpha}}_t) + \dot{\mathrm{Z}}_t.(\hat{\boldsymbol{\alpha}}_t - \tilde{\boldsymbol{\alpha}}_t) \\ &= \mathrm{Z}_t(\hat{\boldsymbol{\alpha}}_t) \end{aligned}$$

where the approximation error is reduced to zero.

Estimation

We estimate the unknown parameters of model (9) by numerical maximisation of the log-likelihood function yielded by the prediction error decomposition (see Harvey (1989)). The latter is evaluated when the SLC algorithm has converged and uses quantities from the Kalman filter. More specifically we use the log-likelihood function of Durbin and Koopman (2012) which is suited for a diffuse initialisation. The filtered estimates are the best prediction of the state vector and its variance at time t using information up to period t, respectively $E(\boldsymbol{\alpha}_t|\boldsymbol{y}_1, ..., \boldsymbol{y}_t)$ and $E(P_t|\boldsymbol{y}_1, ..., \boldsymbol{y}_t)$.

We obtain the best prediction of the state vector and its variance using the entire information set, respectively $E(\boldsymbol{\alpha}_t|\boldsymbol{y}_1, ..., \boldsymbol{y}_N)$ and $E(P_t|\boldsymbol{y}_1, ..., \boldsymbol{y}_N)$, through the Kalman smoother. Those are sometimes referred to as smoothed estimates. The Kalman smoother is a backward recursion algorithm that makes use of the Kalman filter output when the parameters are set to their maximum likelihood estimates.

We use the algorithms of Koopman and Durbin (2000), which are univariate algorithms for multivariate models. With these missing values are accommodated by changing the dimension of the observation vector. Hence, we can use information in the covariates even in months where the VAT data are missing.

The Kalman filter is initialised with the mean $\boldsymbol{a}_1$ and variance P_1 of the initial state vector. We initialise the trend, slope and seasonal components with an exact diffuse initialisation, that is to say with an arbitrary mean and an infinite variance, because they are not stationary. The irregular component, on the other hand, is a stationary process and needs to be initialised with its stationary mean and variance. We assume that the mean of the autoregressive process is zero while its stationary variance depends on the autoregressive order.

Following the method of Durbin and Koopman (2012), we set $\boldsymbol{a}_1 = \boldsymbol{0}$ and $P_1 = \kappa P_\infty + P_*$ where $\kappa \to \infty$ and P_∞ and P_* are $(m \times m)$ diagonal matrices with m indicating the number of states. The diagonal elements of P_∞ are equal to one for non-stationary state elements and equal to zero otherwise. The diagonal elements of the matrix P_* indicate the initial variances of the stationary states and are zero for non-stationary states.

Since we assume stationary autoregressive models we need to constrain the autoregressive parameters accordingly. To do so we reparametrise the autoregressive models in a way that leaves the maximum likelihood estimation unconstrained. Appendix C provides the details of the reparametrisations. The white noise, AR(1) and AR(2) models provide broadly similar results, with a marginal increase in the log-likelihood from the white noise to the AR(1) model. We favour a parsimonious framework which leads us to retain the white noise specification for the irregular component.

To complete the presentation of the model and estimation strategy we now describe how we detect and treat outlying observations in the covariate series.

4.7 Detection and treatment of outlying observations

Economic time series can be subject to outlying observations, that is observations that are not explained by the model appropriately. In the case of the VAT data there can notably be reporting errors. These are captured by the disturbances in the seasonal component in our model.[5] Other types of outlying observations include extraordinary turnover figures and structural breaks. These can affect both series but, because of the unobservability of monthly VAT figures and the noise embedded in them, they are detectable only in the covariate series.[6]

Outliers can affect the disturbances in both the trend and the irregular component. Consequently, they can affect the estimated relationship between the interpolands and the covariates, and, for this reason, it is important to account for them. The outlier identification in state space models is done through the analysis of the auxiliary residuals. Important variations in the standardised irregular disturbances can suggest the presence of extraordinary returns, and outliers in the standardised trend disturbances indicate structural breaks. The standardised state disturbances are given by

$$\hat{\eta}_t^s = B_t^{\eta}\hat{\eta}_t, \qquad \text{with} \quad B_t^{\eta'} B_t^{\eta} = [\mathrm{Var}(\hat{\eta}_t)]^{-1} \tag{10}$$

where $\hat{\eta}_t$ and $\mathrm{Var}(\hat{\eta}_t)$ are the state disturbances' vector and variance-covariance matrix respectively; both are outputs of the Kalman smoother.

The auxiliary residuals can be seen as a series of two-tailed t-tests with the null hypothesis that the observation or state estimate at time t is not an outlier. Thus, we can define a threshold and admissibility region indicating outliers. We choose a confidence interval of 95% which yields a threshold of +/- 1.96. When detected, outliers are modelled with pulse variables taking the form

$$\begin{aligned} d_{i,t}^h &= 1, \qquad \text{if} \quad t = \tau_i^h \\ d_{i,t}^h &= 0, \qquad \text{otherwise,} \end{aligned} \tag{11}$$

where $i = 1, 2$, and $h = e$, μ, and where τ_i^h corresponds to the month where we reject the null hypothesis that the disturbance of the h's component at month t is not an outlier. We can then extend model (7) by including the pulse variables (11) as distinct states and modifying

[5] For some industries the number of outliers detected can be large compared to the total number of observations we dispose, and accounting for outlying VAT data in these industries produces excessively smooth interpolands. This is because accounting for outliers with dummies is equivalent to treating them as missing values; hence we loose all the information embedded in the observation. The smoothness of the interpolated series suggests that the outliers detected in the VAT figures are still partially useful in identifying the monthly changes in the interpolands; consequently, they should not be omitted from the estimation.

[6] We can observe steep changes in the trend but are not able to identify clearly in which month most of the variation is happening. Nevertheless, these level shifts should not take more than three months before being fully accounted for in the estimation.

the trend and irregular state equations accordingly:

$$\begin{aligned}
\Phi(L)\boldsymbol{e}_{t+1} &= \sum_{i=0}^{D^e} \boldsymbol{d}^e_{i,t}\boldsymbol{\delta}^e_{i,t} + \boldsymbol{\kappa}_t, & \boldsymbol{\kappa}_t &\sim \mathrm{N}(0, \Sigma_\kappa), \\
\boldsymbol{\mu}_{t+1} &= \boldsymbol{\mu}_t + \boldsymbol{\nu}_t + \sum_{i=0}^{D^\mu} \boldsymbol{d}^\mu_{i,t}\boldsymbol{\delta}^\mu_{i,t} + \boldsymbol{\xi}_t, & \boldsymbol{\xi}_t &\sim \mathrm{N}(0, \Sigma_\xi), \\
\boldsymbol{\delta}^h_{t+1} &= \boldsymbol{\delta}^h_t, \qquad h = e, \mu,
\end{aligned} \tag{12}$$

for $t = 1, ..., N$, where $\boldsymbol{d}^h_t = (\boldsymbol{d}^h_{1,t}, ..., \boldsymbol{d}^h_{D^h,t})'$, $\boldsymbol{\delta}^h_t = (\boldsymbol{\delta}^h_{i,t}, ..., \boldsymbol{\delta}^h_{D^h,t})'$ and D^h is the total number of distinct pulse variables for the state component h. After the estimation we can analyse the statistical significance of the pulse variables.

The state space matrix form of model (12) is built from the matrix form of the SUTSE model (7) with the following extensions to include the pulse variables: $Z_{2,t} = (1, 0, 1, 0, ..., 0, 1, 0, h^a_{2,t}, \mathbf{0}')$; $\boldsymbol{\alpha}_{2,t} = (\mu_{2,t}, \nu_{2,t}, \gamma_{2,t}, ..., \gamma_{2,t-10}, e_{2,t}, e_{2,t-1}, \beta_{2,t}, \boldsymbol{\delta}^{\mu'}_t, \boldsymbol{\delta}^{e'}_t)'$, $T[20, 33 + j] = 1$ for $j = 0$ to D^μ; $T[30, 33 + D^\mu + j] = 1$ for $j = 0$ to D^e.

5 An application: Land transport and transport service via pipelines

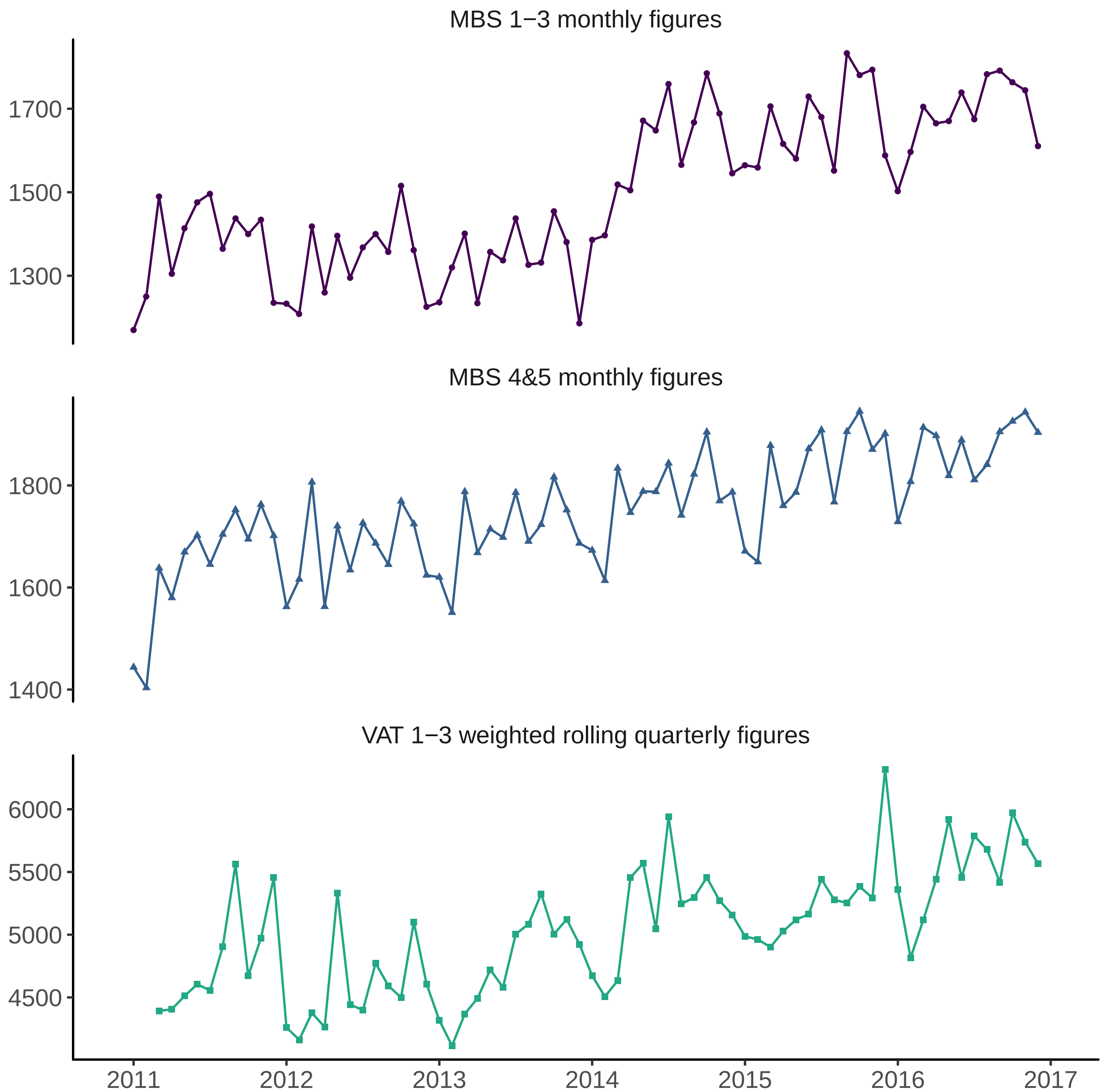
We illustrate our state space approach to disaggregate temporally the VAT data for 'Land transport and transport services via pipelines', industry 493T495. Turnover of firms in size bands one to three accounts to approximately 45% of total turnover in this industry, and it has a weight of 1.5% in overall gross value added, making it one of the most important industries in our sample.

Figure 3 shows the industry's raw data. These include the VAT rolling quarterly data for bands one to three subject to temporal disaggregation, the MBS data for bands four and five that can be used as an indicator series and the MBS data for bands one to three. We aggregate the latter to construct a synthetic rolling quarterly series that we can subsequently disaggregate temporally; the resulting estimates can hence be compared to the true series. In this way we can benchmark the efficiency of our method when applied to clean data. This allows us to understand better the effect that the noise in the VAT data has on the estimation.

We begin by testing our temporal disaggregation method with the synthetic series using a univariate version of our model - the latter is simply derived from model (7) using a single series. We proceed by applying the same model to the VAT data and compare the two sets of results. We then show how the multivariate specification can improve the estimation and help us retrieve some of the volatility embedded in the interpolands. We conclude this section by comparing the seasonally adjusted interpolands with the MBS estimates for bands one to three.

We use two types of information criteria as measures of goodness of fit of the different models and comparison metrics. The first set of criteria are the log-likelihoods of the fitted models, which inform us of their explanatory and prediction capacities. However, there are two caveats with this measure. First, the log-likelihood can only increase with the number

Figure 3: Raw data from industry 493T495, non-seasonally adjusted series, in £million



of parameters estimated; this is not satisfactory for us because the number of parameters varies across models. Second, the log-likelihood measures different things with univariate and multivariate models. With the former it captures the goodness of fit of the model with the VAT series, while with the multivariate model it captures the goodness of fit with both the VAT and MBS 4-5 data; hence it is difficult to know whether the improvement in the log-likelihood can be attributed to a better explanation of the VAT data. A common solution to the first problem is to use the Akaike information criterion (AIC) or the Bayesian information criterion (BIC), but these would not alleviate the issue related to the multivariate specification.

An issue we have become aware of when estimating our models is the excessive smoothness of the monthly interpolands. This comes from the difficulty in separating the noise in the VAT figures from the monthly changes in the interpolands. This leads us to use a measure of volatility in the interpolands as a second indicator. Specifically, we calculate the squared log changes in the seasonally adjusted monthly figures:

$$\text{RSSQ}_i = \sqrt{\sum_{t=2}^{n}(\widehat{x}_{i,t} - \widehat{x}_{i,t-1})^2}$$

where $x_{i,t}$ are the monthly seasonally adjusted estimates in logs, and where $i = 1$ for the interpolands and $i = 2$ for the covariates when estimating multivariate models. Obviously we have no particular view on the value this statistic 'ought' to take, but it is helpful to be able to compare it with the comparable figure for the MBS, in order to indicate whether our interpolated series is more or less volatile than the MBS data.

When analysing the synthetic series we can use an additional criterion. We can directly compare the seasonally adjusted monthly estimates with the seasonally adjusted synthetic series, which we do by defining the following root-mean-square interpolation error:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\exp\widehat{x}_{1,t} - \exp x_{1,t})^2}$$

where $\exp x_{1,t}$ is the 'true series', that is the seasonally adjusted synthetic series. However, the raw data we use to construct the synthetic series are not seasonally adjusted. To remedy this issue we seasonally adjust the raw synthetic series using the same seasonal model that we apply to the interpolated figures (including the Easter effect). This means that the seasonal adjustment of the synthetic series varies depending on the model specification we use for the interpolation. Thus, we can directly observe the efficiency of the interpolation procedure because both the synthetic and the interpolands are subject to the same seasonal adjustment method.

We perform our analysis in R and use the functions from the KFAS package (Helske, 2017) for the Kalman filter and smoother with exact diffuse initialisation and the prediction error decomposition. The Kalman filter and smoother algorithms are based on the univariate approach to multivariate models.

5.1 Univariate analysis with a synthetic series

We begin our empirical analysis by studying the synthetic series. Thus we are able to analyse the goodness of fit of our baseline model specification with clean data. We disaggregate temporally the synthetic series using the nonlinear univariate model.

The results of the estimation are shown in table 3 alongside the information criteria, while the estimates are plotted in Figure 4. The top-left graph compares the seasonally adjusted monthly interpolands to the true series. We can see that the interpolands are close to the underlying true series, and this is confirmed with a low root-mean-square interpolation error. With a root sum of squared log changes (RSSQ criterion) of 0.418, the interpolands are slightly smoother than the true series which exhibits a root sum of squared log changes of 0.189. The state space estimation typically generates a smoother series.

The estimated variance in the trend disturbances suggests a time-varying trend, while the very low estimated variances for the slope and seasonal components suggest they are deterministic. For the latter this is an outcome of a small sample from which it is difficult to disentangle the seasonal disturbances from the aggregation of the monthly changes. We can also observe that the first twelve seasonal disturbances are equal to zero. This comes from the form through which we model the seasonal effects and it does not pose a problem for the estimation.

Table 4 shows that the Easter effect is negative. Yet, the standard error associated with the estimate is large and allows us to accept the hypothesis that the Easter effect is zero. This is the case for all the subsequent estimation results. However, an important caveat regarding the t-tests of the Easter effect is relatively low number of years that we have available (we only observe six Easter periods).

The results of the estimation with the synthetic series suggest that the univariate state space model and estimation procedure is satisfactory for disaggregating temporally rolling quarterly turnover figures that are not seasonally adjusted. However, unlike the VAT data, the synthetic data are not noisy. We now proceed with the estimation of the VAT quarterly series of the 'Land transport and transport service via pipelines' industry.

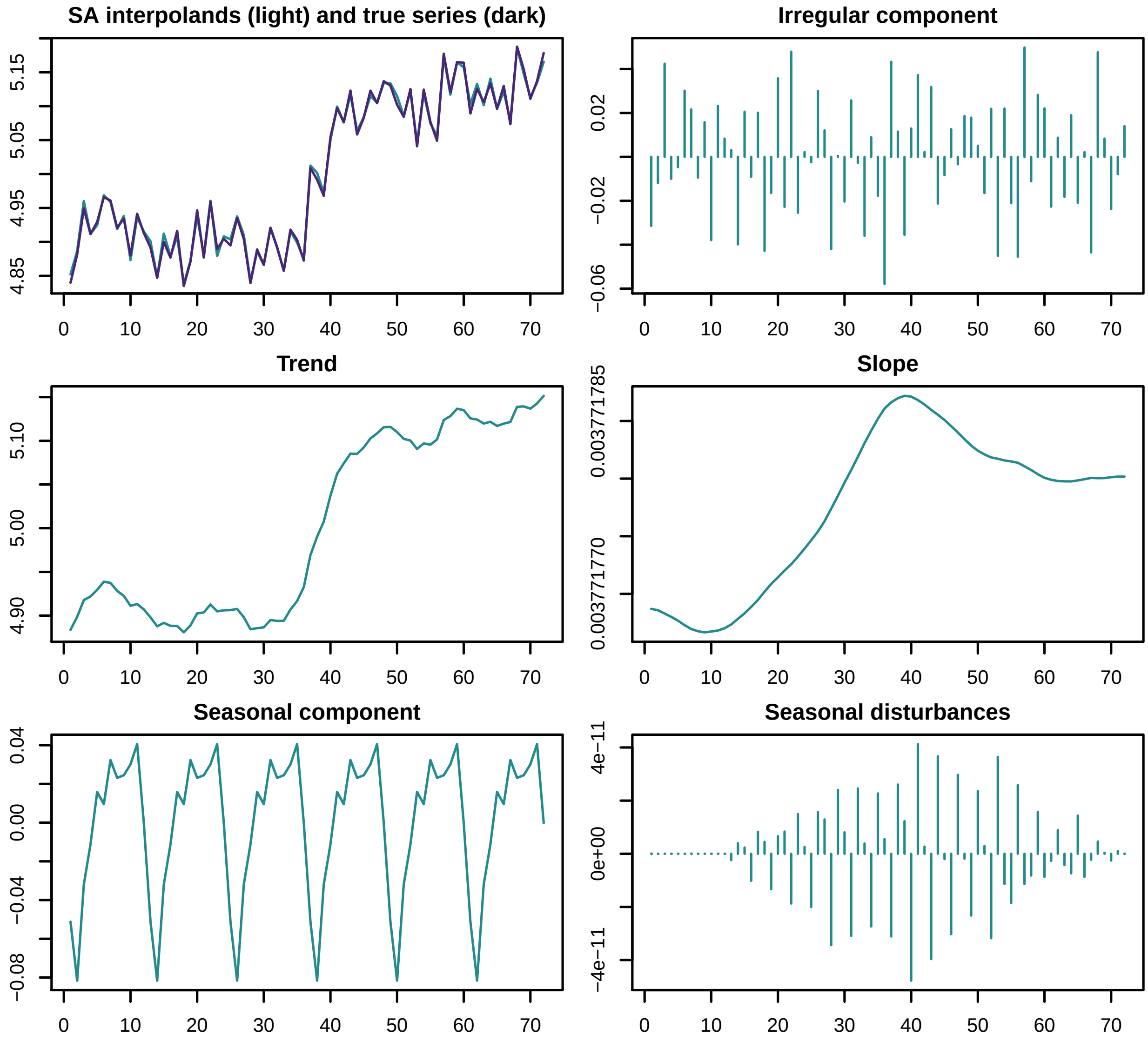
Table 3: Estimation of the univariate model with synthetic data

$\widehat{\sigma}^2_\xi$	$\widehat{\sigma}^2_\omega$	$\widehat{\sigma}^2_\zeta$	$\widehat{\sigma}^2_\kappa$	$RSSQ$	LL	RMSE
0.000436	2.98e-14	4.69e-12	0.00123	0.418	133	0.00578

Table 4: Estimates of Easter effect

Coefficient	Std. error	t-value	p-value
-0.00638	0.00899	-0.71	0.478

Figure 4: Unobserved components' estimates with the univariate model and synthetic data



5.2 Univariate analysis with the VAT series

Having demonstrated the goodness of fit of the univariate state space model with a synthetic series, we now apply it to the quarterly VAT figures. The results of the estimation are presented in tables 5 and 6, and the state estimates are shown in figure 5. These display two important differences from the estimation of the synthetic series.

First, the estimated interpolands of the VAT figures are smoother than those of the synthetic series. This is reflected by a smaller root sum of squared log changes in the seasonally adjusted figures. But the variance estimate of the irregular component is significantly larger than with the synthetic series. This is because of the serial correlation in the irregular component estimates, which is clearly visible from their plot. The serial correlation they exhibit implies that their estimated variance is many times larger than their effective variance.

The second important distinction, which is linked to the first, is the volatility in the estimates of the seasonal effects. The noise in the data is captured in the seasonal disturbances, but it seems that the latter also capture some of the volatility in the interpolands. A direct consequence of this is that the variance of the seasonal disturbances is considerably larger with the VAT series than with the synthetic series. Overall, we can see from the lower log-likelihood yielded by the estimation of the VAT series that the model is less efficient in estimating the VAT data than the synthetic series.

The estimated variance in the slope disturbances is close to zero, and, in this case, it is usually better to constrain the slope to be deterministic. Yet, it could be that that this a particular feature of the univariate model; hence we decide to let the slope vary over time in the subsequent models. As with the synthetic series, the Easter effect is insignificant, which is consistent with the fact that both series should represent the same population.

Overall the estimation of the VAT rolling quarterly series suggests that the univariate model is not efficient in separating the noise in the data from the underlying monthly changes in the interpolands. To remedy this issue we proceed with a multivariate estimation, where the MBS for the largest businesses is used as an indicator series for the month-on-month changes in the seasonally adjusted interpolands.

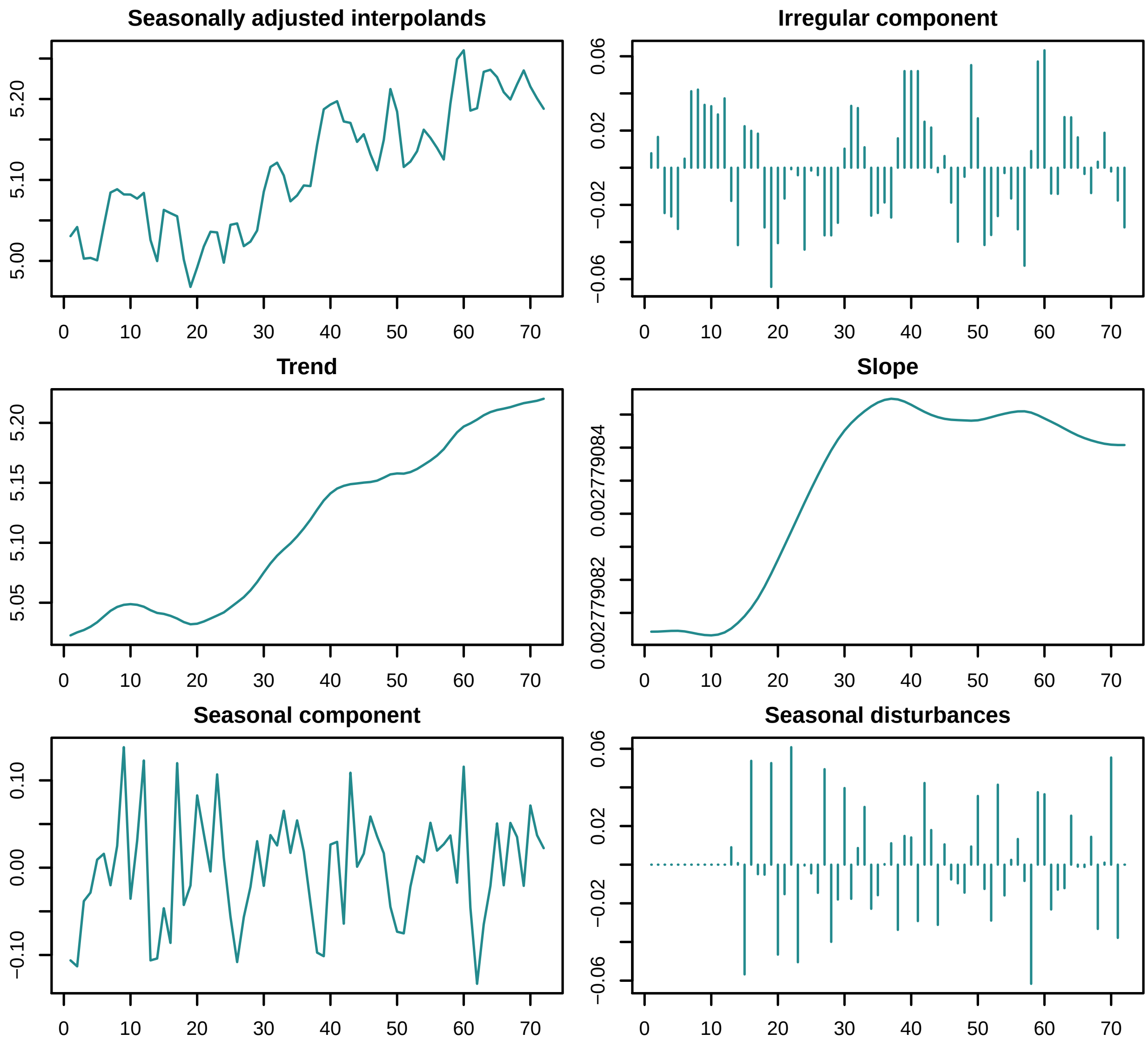
Table 5: Estimation of the univariate model with VAT data

$\widehat{\sigma}^2_\xi$	$\widehat{\sigma}^2_\omega$	$\widehat{\sigma}^2_\zeta$	$\widehat{\sigma}^2_\kappa$	$RSSQ$	LL
0.000132	0.00146	6.42e-13	0.00377	0.254	58.5

Table 6: Estimates of Easter effect

Coefficient	Std. error	t-value	p-value
0.0022	0.0367	0.0599	0.952

Figure 5: Unobserved components' estimates with the univariate model and VAT data



5.3 Multivariate analysis with the VAT and covariate series

We know from the univariate analysis of the synthetic and VAT series that, although our model specification is adapted to the rolling nature of the data, the noise in the VAT series poses is a problem for the identification of the monthly changes in interpolands. A possible solution to this is to use the MBS for the largest businesses as an indicator series in a multivariate framework. Hence, we estimate the multivariate model (7) where we let the disturbances in the irregular components be contemporaneously correlated. The results of the estimation are presented in table 7 and 8, and figure 6 plots the interpolands and covariates alongside their irregular variations.

The estimated correlation coefficient between the irregular disturbances in the seasonally adjusted interpolands and covariates is positive and equal to 0.309. As a result, the estimated variance in the irregular disturbances of the interpolands has increased from 0.00377 to 0.00408, and the root sum of squared log changes has increased from 0.254 to 0.331. The greater volatility in the intepolands can also be observed in Figure 7 where we compare the estimates from the univariate and multivariate models. We conclude that the covariate series is helpful in identifying month-on-month changes in the seasonally adjusted interpolands.

We proceed with an analysis of outliers in the MBS series.

Table 7: Estimation of the multivariate model with VAT data

$\widehat{\sigma}^2_{1\xi}$	$\widehat{\sigma}^2_{1\omega}$	$\widehat{\sigma}^2_{1\zeta}$	$\widehat{\sigma}^2_{1\kappa}$	$RSSQ_1$
8.58e-05	0.00149	2.82e-11	0.00408	0.331
$\widehat{\sigma}^2_{2\xi}$	$\widehat{\sigma}^2_{2\omega}$	$\widehat{\sigma}^2_{2\zeta}$	$\widehat{\sigma}^2_{2\kappa}$	$RSSQ_2$
4.57e-05	0	1.46e-14	0.000642	0.292
$\hat{\rho}_\kappa$	LL			
0.309	166			

Table 8: Estimates of Easter effects

series	Coefficient	RMSE	t-value	p-value
Interpolands	-0.00498	0.0369	-0.135	0.893
Covariate	-0.0249	0.0177	-1.4	0.161

Figure 6: Unobserved components' estimates with the multivariate model with VAT 1-3 and MBS 4&5 series

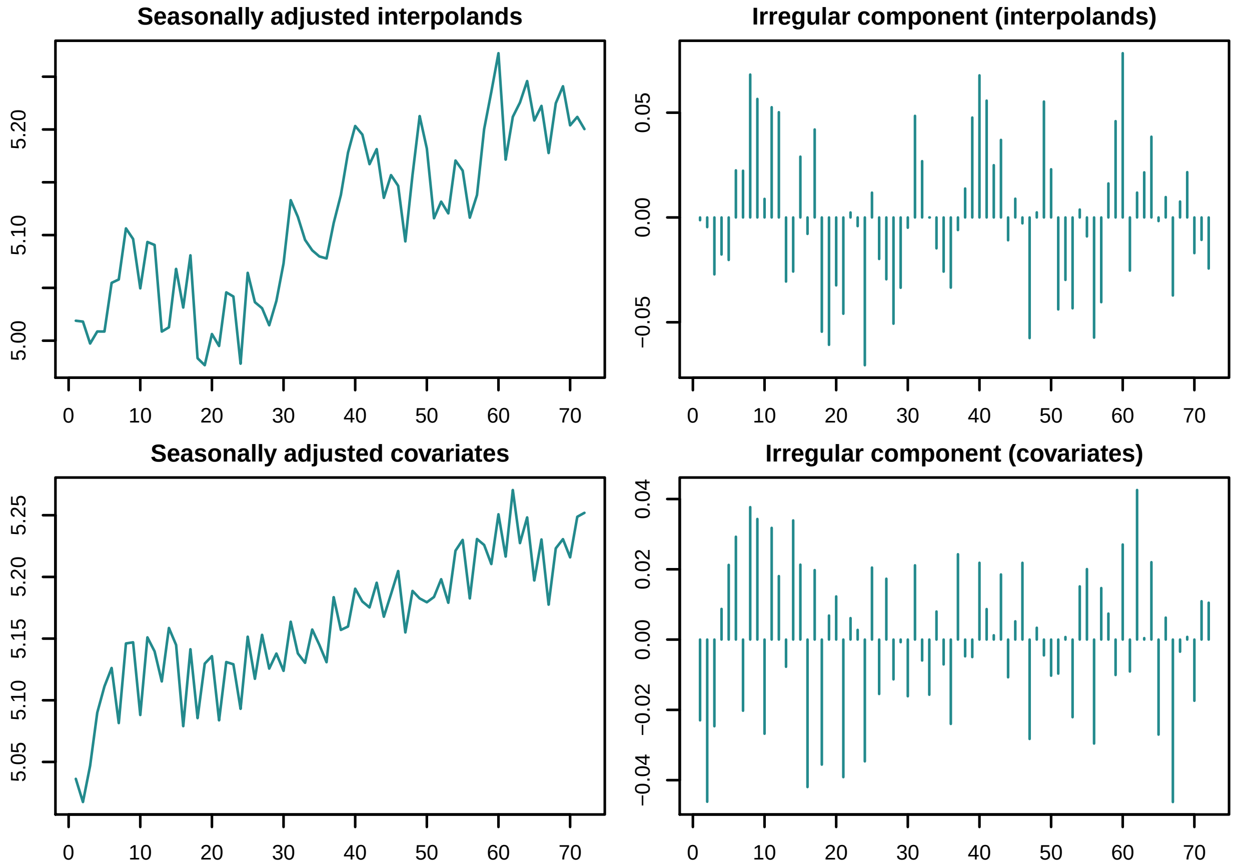
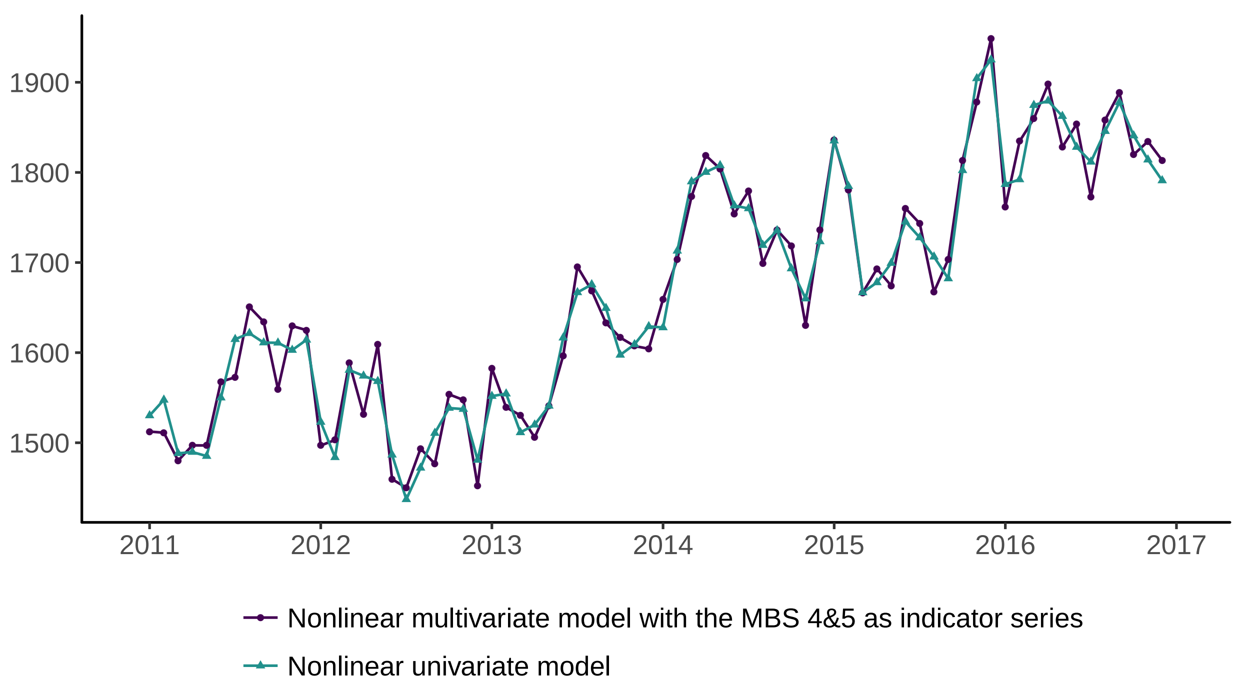


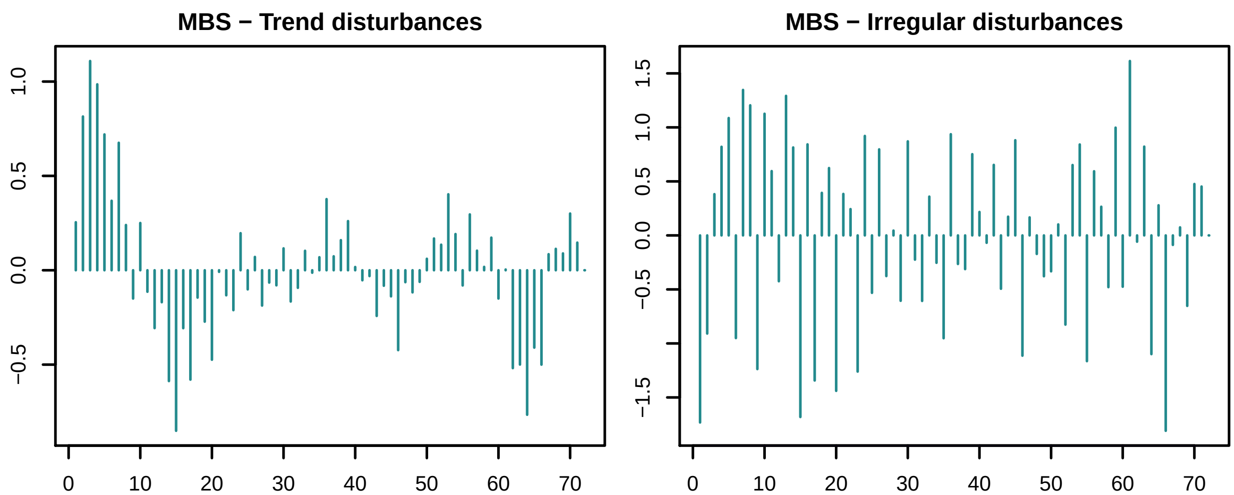
Figure 7: Seasonally adjusted interpolands, in £million



5.4 Treatment of outliers and structural breaks

The estimation of the multivariate model can be enhanced by accounting for outliers in the series. Notably, treating structural breaks and outliers in the MBS can improve the estimation of the correlation between the irregular components. To detect the outliers we analyse the auxiliary residuals resulting from the estimation of the multivariate model. Figure 8 shows the resulting auxiliary residuals, which can be interpreted as two-tailed t-tests with an admissibility region of +/-1.96. The standardised trend and irregular disturbances in the MBS do not show any potential structural break or extraordinary returns.

Figure 8: Auxiliary residuals



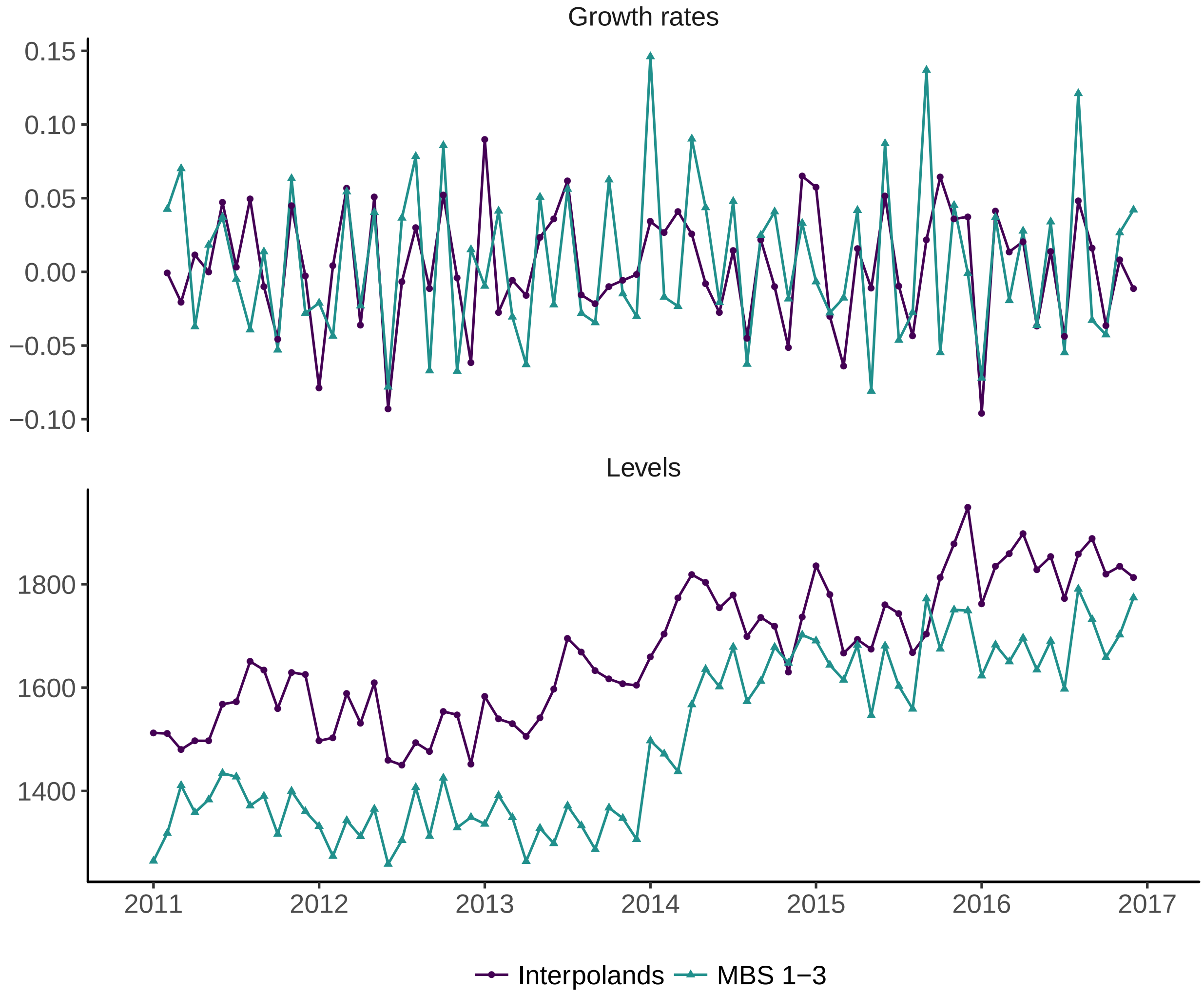
5.5 Comparison with the MBS 1-3 estimates

We are ultimately interested in how the VAT seasonally adjusted monthly estimates compare with the MBS estimates. We seasonally adjust the latter using a univariate model with Easter effect and data in logs. Hence, both series are subject to the same seasonal adjustment method. Figure 9 shows the two series which we plot in levels and in growth rates.

The trends in the VAT and MBS figures diverge significantly from the first half of 2014. This is not surprising; while describing the data we have shown that the VAT and MBS data for bands one to three exhibit different trends, although they represent the same types of businesses. However, the growth rates in the two series do show some similarities, with a correlation between the series in growth rates of 0.48.

With a root sum of squared log changes of 0.331, the VAT monthly seasonally adjusted estimates are less volatile than the MBS estimates which show a root sum of squared log changes of 0.435. There are two explanations for this. On the one hand, the state space estimation typically generates estimates with a lower root sum of squared log changes than the underlying ‘true’ monthly series, and the noise in the VAT data could exacerbate this feature. On the other hand, survey-based estimates are subject to sampling errors which are likely to create some degree of short-term volatility in the MBS estimates.

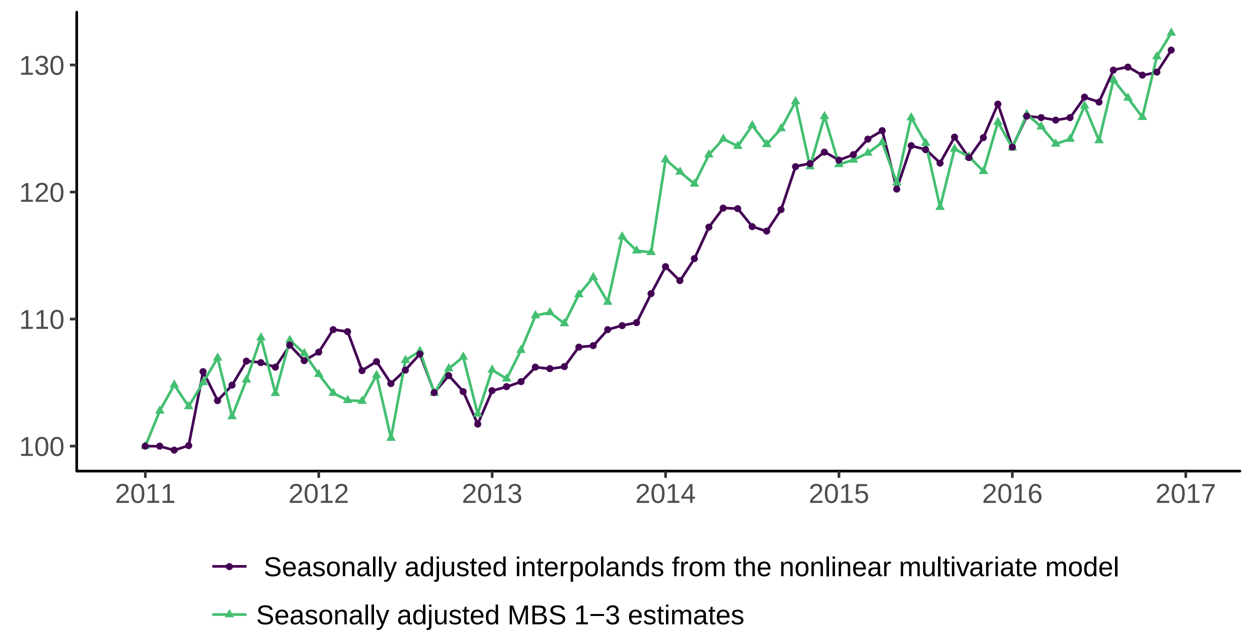
Figure 9: Comparison between the seasonally adjusted interpolands and MBS 1-3 figures, levels in £million



6 Application to all industries

Having illustrated our method with the 'Land transport and transport services via pipelines' industry, we now produce aggregate estimates from the whole set of seventy-five industries. We carry out the estimation with the nonlinear multivariate models with pulse variables that we have identified using a first round of results. We produce those seasonally adjusted monthly series of turnover for each industry, which we then index to March 2011 = 100 and aggregate together using industries' gross value added weights and turnover shares of firms in size bands one to three, as in section 2. Figure 10 compares the resulting series with the seasonally adjusted aggregate figures from the MBS covering small and medium size businesses.

Figure 10: Aggregate monthly turnover estimates, seasonally adjusted figures, index January 2011 = 100



We can see that the MBS and VAT aggregate estimates again follow different trends. Notably, the VAT data point to a slower recovery after the euro area sovereign debt crisis.[7] That is not surprising; we have shown when describing the data that the thirteen-month moving averages of the MBS and VAT series diverge.

Figure 11 shows the discrepancy between the aggregated MBS and VAT-based estimates. The discrepancy between the two series reach a peak in Feb 2014 where the VAT series is approximately 8.5 percentage points lower than the MBS series. This implies a discrepancy of a little bit more than 2 percentage points on the output estimates of GDP since our series cover approximately a quarter of gross value added in the economy.

This discrepancy between the MBS and VAT estimates could arise from the distribution of industries' weights. If industries with high weights exhibit relatively high persistent discrepancies, these could appear in the aggregated figures. To answer this question, figure 12 plots the average log differences between the MBS VAT-based estimates, for each month across all industries. The discrepancies do not average to zero.

Taking this divergence as given, the correlation between the series in growth rates is 0.51. This indicates some similarities between the two series, but, since they should represent the population of businesses, we would expect the correlation to be higher. To understand this divergence it would be helpful to analyse the data at a firm level.

Additionally, the VAT-based turnover aggregated estimates are less volatility than the MBS aggregated estimates, with a root sum of squared log changes of 0.132 for the VAT

[7] For ten industries the MBS and VAT estimates do not cover exactly the same population, but the aggregated series still diverge if we exclude them.

Figure 11: Log differences between the MBS and VAT aggregated estimates

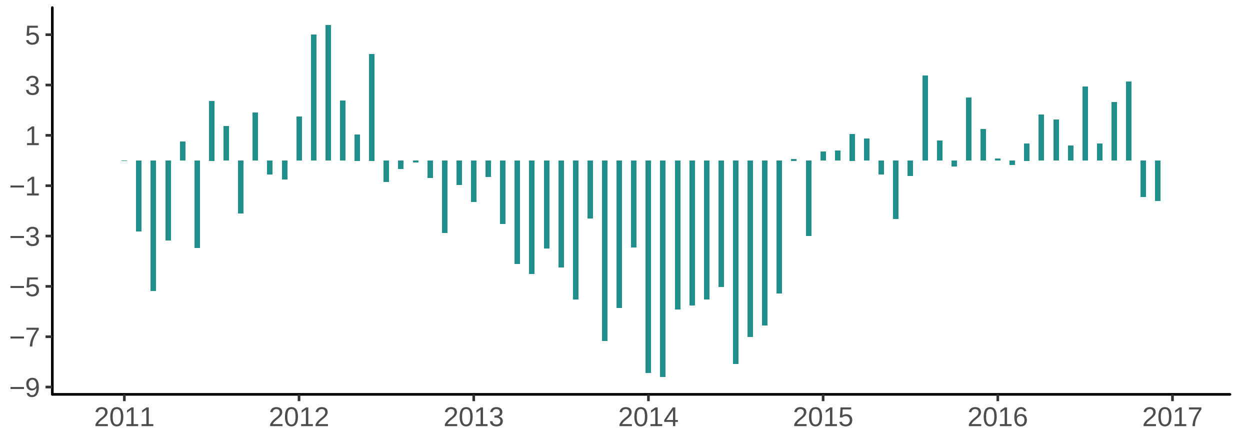
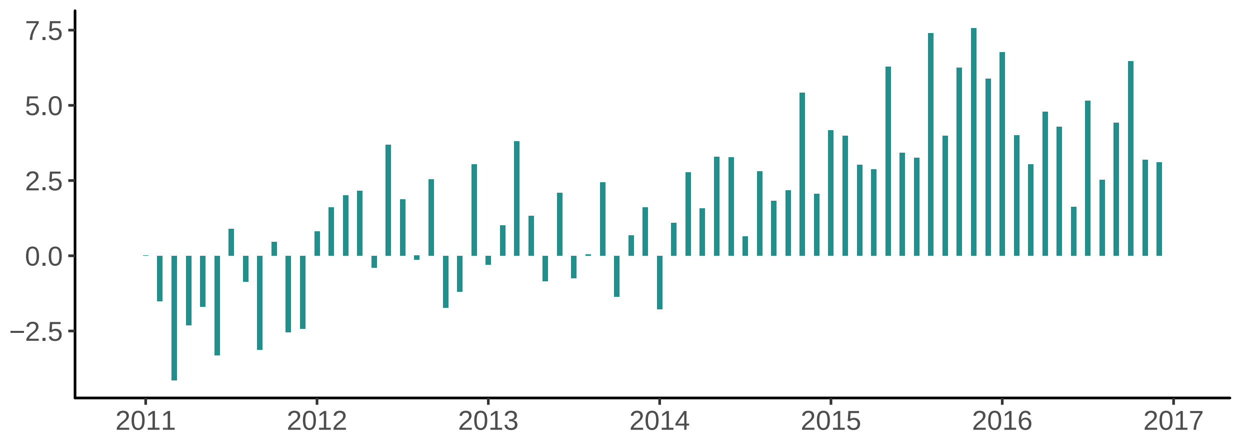


Figure 12: Average log differences between the MBS and VAT estimates across industries



estimates compared to 0.212 for the MBS estimates. That is what we find at an industry level (as illustrated with the example industry), and this feature is transmitted to the aggregated figures.

7 Conclusion

In this work we have explored how business turnover figures provided by VAT returns could be used to replace the MBS covering small and medium size businesses (firms in size bands one to three). This requires to disaggregate the quarterly VAT-based turnover figures into monthly estimates. The rolling nature of the data, as well as the noise and the dynamic seasonality they exhibit, imply that least-squares techniques are not a viable solution for the estimation. We have consequently developed state space methods designed to accommodate these particular features of the VAT series. These techniques are efficient at filtering and temporally disaggregating the data and provide a flexible framework to make use of an indicator series, the MBS for the largest businesses in our case, in identifying monthly movements in the interpolands. In addition, we use a nonlinear model to handle data in logarithms; we therefore account for potential heteroskedasticity in the irregular variations and proportional seasonal effects.

We have illustrated our state space framework with the 'Land transport and transport service via pipelines' industry as a case study. Empirical results from the univariate model suggest that our model specification is satisfactory when applied to clean data, but the noise in the VAT series complicates the identification of the monthly movements in the interpolands. This issue can be alleviated by using the MBS for the largest businesses as an indicator series in a multivariate framework.

Using the nonlinear multivariate model, we have proceeded by estimating aggregated figures of turnover covering approximetly a quarter of gross value added in the economy. We have compared the interpolands with the estimates of turnover derived from the MBS covering small and medium size businesses that the VAT data could potentially replace. The estimates from the VAT data are less volatile, and, interestingly, show a different time profile, although they relate to the same population of businesses.

Further work is required to understand if this divergence between the MBS and VAT estimates is due to conceptual differences in what the VAT returns and MBS measure, or if they are due to sampling or reporting errors. The estimates of turnover derived from the MBS data are subject to revisions once data on expenditure and income are available. The latter are also derived from tax data. It would be particularly interesting to investigate whether the discrepancies between the VAT and MBS estimates are early indicators of those revisions.

Appendix A List of industries

Table 9: SIC 2007 codes and desription of industries, part 1/2

SIC 2007 code	Description
07_08	Mining and quarrying
101	Processing and preserving of meat, production of meat products
102_3	Processing and preserving of fish, crustaceans, fruit and veg
107	Manufacture of bakery and farinaceous products
108	Manufacture of other food products
109	Manufacture of prepared animal feeds
1101T1106	Manufacture of alcoholic beverages
1107	of soft drinks; prod of mineral water and other bottled waters
13	Manufacture of textiles
14	Manufacture of wearing apparel
15	Manufacture of leather and related products
16	Man of wood and prod of wood and cork, Excl furniture
17	Manufacture of paper and paper products
18	Printing and reproduction of recorded media
203	of paints,varnishes and similar coatings, printing ink and mastics:
204	Man of soap and detergents,cleaning products ,perfume
205	Manufacture of other chemical products
20A	Man of ind gases, inorganic chemicals and fertilisers
20C	Manufacture of dyestuffs, agro-chemicals- 20.12/20
21	Manufacture of basic pharmaceutical products and preparations
22	Manufacture of rubber and plastic products
235_6	Man of cement, lime,plaster and art of concrete,cement and plaster:
23OTHER	Man of glass, clay, porcelain, ceramic, stone products
25OTHER	Man of fabricated metal products, excl weapons and ammo
26	Manufacture Of Computer, Electronic and Optical Products
27	Manufacture of electrical equipment
28	Manufacture of machinery and equipment N.E.C.
29	Manufacture of motor vehicles, trailers and semi-trailers
301	Building of ships and boats
303	Manufacture of air, spacecraft and related machinery
30OTHER	Manufacture of other transport equipment- 30.2/4/9
31	Manufacture of furniture
32	Other manufacturing
3315	Repair and maintenance of ships and boats
3316	Repair and maintenance of aircraft and spacecraft
33OTHER	Rest of repair; Installation - 33.11-14/17/19/20
37	Sewerage

Table 10: SIC 2007 codes and description of industries, part 2/2

SIC 2007 code	Description
38_39	Waste collection, treatment and disposal activities; materials recovery; and remediation activities and other waste management services
41	Construction of buildings
42	Civil engineering
43	Specialised construction activities
45	Wholesale, retail trade and repair of motor vehicles and motorcycles:
46	Wholesale trade, excl motor vehicles and motorcycles
493T495	Land transport and transport services via pipelines
52	Warehousing and transport support activities
55	Accommodation
56	Food and beverage service activities
58	Publishing activities
59	Motion picture, video and television production activities
60	Programming and broadcasting activities
61	Telecommunications
62	Computer Programming, Consultancy and Related Act
63	Information service activities
683	Real estate activities on a fee or contract basis
691	Legal activities
692	Accounting, bookkeeping and auditing activities; tax consultancy
70	Activities of head offices; management consultancy activities
71	Architectural and engineering act; technical testing and analysis
72	Scientific research and development
73	Advertising and market research
74	Other Professional, Scientific and Technical Act
75	Veterinary activities
77	Rental and leasing activities
78	Employment activities
79	Travel agency, tour operators and other reservation related act
80	Security and investigation activities
81	Services to buildings and landscape activities
82	Office admin, office support and other business support act
85	Education
86	Human health activities
90	Creative arts and entertainment activities
91	Libraries, Archives, Museums and Cultural Activities
93	Sports activities, amusement and recreation activities
95	Repair Of Computers and Personal and Household Goods
96	Other personal service activities

Appendix B Borrowing the seasonal pattern from the indicator series: the common seasonality model

While not being able to extract monthly seasonal effects is not a problem for the estimation of monthly seasonally adjusted figures, we are not able to retrieve monthly estimates that are not seasonally adjusted. One way to get round this issue is to assume that the seasonality in the interpolands is proportional to the seasonality in the indicator series. We can achieve this in the multivariate model by assuming that the seasonal component in the interpolands and covarariates share the same disturbances and that the interpolands' fixed seasonal term is zero. This is equivalent to assuming *identical seasonal components*, which are a restricted form of *common seasonal components* (see Harvey and Koopman (1997)). Moauro and Savio (2005) also study this approach in the context of temporal disaggregation. This is achieved by setting the observation equations of the multivariate model as

$$
\begin{aligned}
y_{1,t} &= x_{1,t} + x_{1,t-1} + x_{1,t-2} + \gamma_{1,t}^{stagger} + \gamma_{1,t-1}^{stagger} + \gamma_{1,t-2}^{stagger} + \beta_{1,t} h_{1,t}^{a}, \\
y_{2,t} &= x_{2,t} + \gamma_{2,t} + \beta_{2,t} h_{2,t}^{a}, \\
\gamma_{1,t} &= \Phi \gamma_{2,t} \\
\gamma_{1,t+1}^{stagger} &= \sum_{j=1}^{2} \gamma_{1,t+1-j}^{stagger} + \omega_t^{stagger}, \qquad \omega_t^{stagger} \sim N(0, \sigma_{\omega^s}^2),
\end{aligned}
\tag{13}
$$

for $t = 1, ..., N$, where $\gamma_{2,t}$ follows a trigonometric or dummy seasonal model and where γ_t^s is the stagger bias. For simplicity we use data in levels in this appendix, therefore the model is linear. The scalar Φ can be estimated via maximum likelihood estimation or simply fixed to a predetermined value. The observation equation of the VAT series now exhibits two lags of the seasonal effects; hence we need to augment the state vector by twelve components if we use the trigonometric model. No augmentation is needed if we use the dummy seasonal model.

Since we can now estimate monthly seasonal effects through the seasonality in the indicator series, we can also identify and estimate the stagger bias γ_t^s. We do so by modelling the latter as seasonal dummy variables of frequency three and adding it to the observation equation. It is now the stagger bias disturbances that capture the noise in the data instead of the seasonal disturbances, because the latter are proportional to the seasonal disturbances in the MBS series for strata 4 and 5.

In this appendix we analyse the implications of applying the seasonality in the covariates to the interpolands. The motivation behind this is to retrieve monthly seasonal effects, which we cannot do otherwise. We use the 'Land transport and transport services via pipelines' industry as an example.

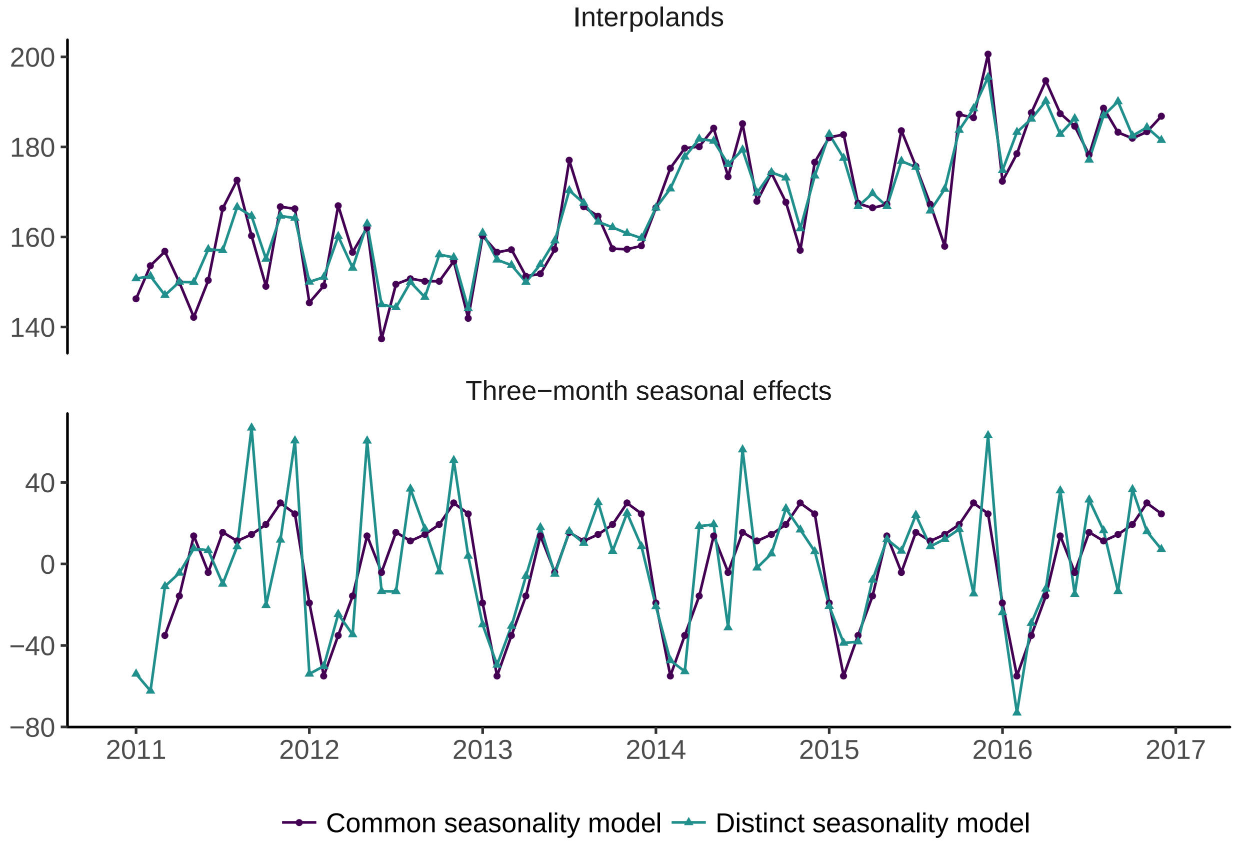
For the estimation of the common seasonal model we multiply the covariates series by ten. This insures that the covariates' seasonal effects are greater than those of the interpolands, and we can in this way constrain the parameter Φ to lie between 0 and 1, which tends to improve the estimation. The results of the estimation of the common seasonal model are presented in table 11, where they are compared to the normal SUTSE model with distinct seasonal components. The estimated interpolands and seasonal effects are shown in figure 13. The estimates of the seasonal effects from the common seasonal model need to be aggregated over three months to be compared with the VAT seasonal estimates.

The results show a clear decrease in the log-likelihood of the fitted model with common seasonal component, which is associated with an increased volatility in the interpolands.

Table 11: Estimation results of the SUTSE models

Parameters	Distinct seasonality model	Common seasonality model
$\widehat{\sigma}^2_{1\xi}$	1.61	1.17e-05
$\widehat{\sigma}^2_{1\omega}$	405	124 (stagger bias component)
$\widehat{\sigma}^2_{1\zeta}$	2.85e-07	4.84e-12
$\widehat{\sigma}^2_{1\kappa}$	131	164
$RSSQ_1$	0.146	0.302
$\widehat{\sigma}^2_{2\xi}$	0.671	0.0118
$\widehat{\sigma}^2_{2\omega}$	0	0
$\widehat{\sigma}^2_{2\zeta}$	2.16e-09	3.22e-26
$\widehat{\sigma}^2_{2\kappa}$	19.8	2290
$RSSQ_2$	0.0848	0.0933
$\hat{\rho}_\kappa$	0.355	0.321
LL	-484	-654
$\hat{\Phi}$		0.235

Figure 13: Estimates of the interpolands and aggregated seasonal effects



Appendix C Autoregressive models for the irregular component

An AR(1) model

The state equation of the irregular component can be rewritten as

$$e_{t+1} = \phi_1 e_t + \kappa_t, \qquad \kappa_t \sim \mathrm{N}(0, \sigma^2_\kappa).$$

We assume that the AR model is stationary and e_t has to be initialised accordingly. Notably, we set

$$e_1 \sim \mathrm{N}(0, \sigma^2_\kappa/(1-\phi_1^2)).$$

Because we assume a stationary process, the maximum likelihood estimation for ϕ_1 has to be carried out in a constrained parameter space; we notably need $|\phi_1| < 1$ for the AR(1) model to be stationary. However, we want to keep the maximum likelihood estimation unconstrained, and one way to do so is to define

$$\phi_1 = \frac{e^\rho - e^{-\rho}}{e^\rho + e^{-\rho}},$$

and minimise the negative log-likelihood function with respect to ρ.

An AR(2) model

The state equation of the irregular component can be rewritten as

$$e_{t+1} = \phi_1 e_t + \phi_2 e_{t-1} + \kappa_t, \qquad \kappa_t \sim \mathrm{N}(0, \sigma_\kappa^2).$$

We set the distribution to the initial autoregressive component to

$$e_1 \sim \mathrm{N}(0, \frac{(1-\phi_2)\sigma_\kappa^2}{(1+\phi_2)((1-\phi_2)^2 - \phi_1^2)}).$$

For the AR(2) model to be stationary we need the root of the polynomial

$$1 - \phi_1 x - \phi_2 x^2 = 0$$

to lie outside the unit circle. However, it is easier to work with

$$\lambda^2 + a\lambda + b = 0, \qquad \text{where} \quad \lambda = 1/x, \quad a = -\phi_1 \quad \text{and} \quad b = -\phi_2. \tag{14}$$

We now need the root of polynomial (14) to lie within the unit circle, which is true if and only if the conditions

$$|b| < 1 \qquad \text{and} \quad |a| < 1 + b$$

are satisfied. From Osborn (1976) we know that these conditions can be enforced by defining

$$\begin{aligned} b &= \frac{\delta}{1+|\delta|}, \\ a &= \frac{\gamma(1+b)}{1+|\gamma|}, \end{aligned} \tag{15}$$

where δ and γ are unrestricted real parameters. This method allows for the existence of both complex and real roots. We now face an unconstrained optimisation problem when maximising the log-likelihood function with respect to δ and γ.[8][9]

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[8] Alternatively, we could reparametrise the autoregressive model using the partial autocorrelations, which is a method that has extensively been studied by Barndorff-Nielsen and Schou (1973), Monahan (1984), Ansley and Kohn (1986) and Marriott and Smith (1992). We expect both methods to yield similar results for an AR(2) model.

[9] We have checked that this procedure yields satisfactory results by running a set of Monte Carlo simulations.

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