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Decomposing Value Added Growth in Explanatory Factors

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Seminar 3

Business School

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Decomposing Value Added Growth Over Sectors into Explanatory Factors

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Summary

- Decompose nominal value added growth over multiple sectors into explanatory factors.

- For a single sector, explanatory factors are

  - efficiency changes,
  - changes in output prices,
  - changes in primary inputs,
  - changes in input prices,
  - technical progress, and
  - returns to scale.

Summary

- Need sector's best practice technology for the two periods under consideration.
- Could use econometric or nonparametric (DEA) techniques
- We use Free Disposal Hull approach – no convexity assumptions
- Our approach has the advantage that it does not involve econometric estimation, and involves only observable data.
- Simple enough to be implemented by statistical agencies
- If efficient in both periods, can use the index number techniques of Diewert-Morrison (1986)/Kohli (1990).
- Address the problem of aggregating over sectors.

Cost Constrained Value Added Function for a Sector

- **Basic Notation**

- A sector produces M net outputs, $y \equiv [y_1,\ldots,y_M]$, using N primary inputs $x \equiv [x_1,\ldots,x_N] \geq 0_N$.

- If $y_m > 0$, then the sector produces the mth net output during period t while if $y_m < 0$, then the sector uses the mth net output as an intermediate input.

- Strictly positive vector of net output prices $p \equiv [p_1,\ldots,p_M] >> 0_M$ and strictly positive vector of input prices $w \equiv [w_1,\ldots,w_N] >> 0_N$

- *Period t production possibilities set* for the sector S^t

Cost Constrained Value Added Function for a Sector

Period t cost constrained value added function:

$$R^t(p,w,x) \equiv \max_{y,z} \{p \cdot y : (y,z) \in S^t; w \cdot z \leq w \cdot x\}$$

$R^t(p,w,x)$ is well defined even if there are increasing returns to scale in production; i.e., the constraint $w \cdot z \leq w \cdot x$ leads to a finite value for $R^t(p,w,x)$.

If (y^*,z^*) solves this constrained maximization problem, then sectoral value added $p \cdot y$ is maximized subject to the constraints that (y,z) is a feasible production vector and primary input expenditure $w \cdot z$ is equal to or less than "observed" primary input expenditure $w \cdot x$.

Cost Constrained Value Added Function for a Sector

Observed value added, $p^t \cdot y^t$, may not equal the optimal value added.

Value added efficiency of the sector during period t:

$$e^t \equiv p^t \cdot y^t / R^t(p^t, w^t, x^t) \leq 1$$

The cost constrained valued added function has some interesting properties. If S^t is a cone, so that production is subject to constant returns to scale, we can show that

$$R^t(p,w,x) \equiv w \cdot x / c^t(w,p)$$

where $c^t(w,p)$ is the unit cost function for producing a unit of value added.

Decomposing Value Added Growth for a Sector into Explanatory Factors

Change in value added efficiency

$$\varepsilon^t \equiv e^t/e^{t-1} = [p^t \cdot y^t/R^t(p^t,w^t,x^t)]/[p^{t-1} \cdot y^{t-1}/R^{t-1}(p^{t-1},w^{t-1},x^{t-1})]$$

If $\varepsilon^t > 1$, then value added efficiency has *improved* going from period t–1 to t whereas it has *fallen* if $\varepsilon^t < 1$.

Decomposing Value Added Growth for a Sector into Explanatory Factors

Follow method of Konüs (1939) and Allen (1949) to define various *families of indexes* that vary only *one* of the four sets of variables, t, p, w and x, between the two periods under consideration and hold constant the other sets of variables.

Family of *output price indexes*:

$$\alpha(p^{t-1},p^t,w,x,s) \equiv R^s(p^t,w,x)/R^s(p^{t-1},w,x).$$

Two alternatives:

$\alpha_L{}^t \equiv \alpha(p^{t-1},p^t,w^{t-1},x^{t-1},t-1) \equiv R^{t-1}(p^t,w^{t-1},x^{t-1})/R^{t-1}(p^{t-1},w^{t-1},x^{t-1})$;

$\alpha_P{}^t \equiv \alpha(p^{t-1},p^t,w^t,x^t,t) \equiv R^t(p^t,w^t,x^t)/R^t(p^{t-1},w^t,x^t).$

Preferred overall measure of output price growth:

$\alpha^t \equiv [\alpha_L{}^t\,\alpha_P{}^t]^{1/2}$

Decomposing Value Added Growth for a Sector into Explanatory Factors

Family of *input quantity indexes*:

$$\beta(x^{t-1},x^t,w) \equiv w \cdot x^t / w \cdot x^{t-1}.$$

$\beta_L{}^t \equiv w^{t-1} \cdot x^t / w^{t-1} \cdot x^{t-1}$;

$\beta_P{}^t \equiv w^t \cdot x^t / w^t \cdot x^{t-1}$.

Preferred overall measure of input quantity growth:

$\beta^t \equiv [\beta_L{}^t \, \beta_P{}^t]^{1/2}$.

Decomposing Value Added Growth for a Sector into Explanatory Factors

Family of *input mix indexes*:

$$\gamma(w^{t-1},w^t,p,x,s) \equiv R^s(p,w^t,x)/R^s(p,w^{t-1},x)$$

More accurate to say that $\gamma(w^{t-1},w^t,p,x,s)$ represents the hypothetical proportional change in cost constrained value added for the period s reference technology **due to the effects of a change in the input price vector** from w^{t-1} to w^t when facing the reference net output prices p and the reference vector of inputs x.

$$\gamma_{LPP}{}^t \equiv \gamma(w^{t-1},w^t,p^{t-1},x^t,t) \equiv R^t(p^{t-1},w^t,x^t)/R^t(p^{t-1},w^{t-1},x^t);$$

$$\gamma_{PLL}{}^t \equiv \gamma(w^{t-1},w^t,p^t,x^{t-1},t-1) \equiv R^{t-1}(p^t,w^t,x^{t-1})/R^{t-1}(p^t,w^{t-1},x^{t-1}).$$

$$\gamma^t \equiv [\gamma_{LPP}{}^t\gamma_{PLL}{}^t]^{1/2}.$$

Decomposing Value Added Growth for a Sector into Explanatory Factors

Family *of technical progress indexes*:

$$\tau(t-1,t,p,w,x) \equiv R^t(p,w,x)/R^{t-1}(p,w,x)$$

$$\tau_L{}^t \equiv \tau(t-1,t,p^{t-1},w^{t-1},x^t) \equiv R^t(p^{t-1},w^{t-1},x^t)/R^{t-1}(p^{t-1},w^{t-1},x^t).$$
$$\tau_P{}^t \equiv \tau(t-1,t,p^t,w^t,x^{t-1}) \quad \equiv R^t(p^t,w^t,x^{t-1})/R^{t-1}(p^t,w^t,x^{t-1}).$$

Recall, if the reference technologies in periods t and t–1 are cones, $R^t(p,w,x) = w \cdot x/c^t(w,p)$ and $R^{t-1}(p,w,x) = w \cdot x/c^{t-1}(w,p)$.

Thus in the case where the reference technology is subject to CRS, these "mixed" indexes of technical progress are independent of x and then true Laspeyres and Paasche type indexes.

Decomposing Value Added Growth for a Sector into Explanatory Factors

Family *of (global) returns to scale measures*:

$\delta(x^{t-1},x^t,p,w,s) \equiv [R^s(p,w,x^t)/R^s(p,w,x^{t-1})]/[w \cdot x^t/w \cdot x^{t-1}]$.

$\delta_L{}^t \equiv \delta(x^{t-1},x^t,p^{t-1},w^{t-1},t-1) \equiv R^{t-1}(p^{t-1},w^{t-1},x^t)/R^{t-1}(p^{t-1},w^{t-1},x^{t-1})]/[w^{t-1} \cdot x^t/w^{t-1} \cdot x^{t-1}]$;

$\delta_P{}^t \equiv \delta(x^{t-1},x^t,p^t,w^t,t) \equiv [R^t(p^t,w^t,x^t)/R^t(p^t,w^t,x^{t-1})]/[w^t \cdot x^t/w^t \cdot x^{t-1}]$.

$\delta^t \equiv [\delta_L{}^t\, \delta_P{}^t]^{1/2}$

Decomposing Value Added Growth for a Sector into Explanatory Factors

Six explanatory growth factors:

1. Change in cost constrained value added efficiency: $\varepsilon^t \equiv e^t/e^{t-1}$

2. Change in output prices: $\alpha(p^{t-1},p^t,w,x,s)$

3. Change in input quantities: $\beta(x^{t-1},x^t,w)$

4. Change in input prices: $\gamma(w^{t-1},w^t,p,x,s)$

5. Changes due to technical progress: $\tau(t-1,t,p,w,x)$

6. Returns to scale measure: $\delta(x^{t-1},x^t,p,w,s)$

Decomposing Value Added Growth for a Sector into Explanatory Factors

Exact decompositions of observed value added:

$p^t \cdot y^t / p^{t-1} \cdot y^{t-1} = \varepsilon^t \alpha_P{}^t \beta_L{}^t \gamma_{LPP}{}^t \delta_L{}^t \tau_L{}^t$;

$p^t \cdot y^t / p^{t-1} \cdot y^{t-1} = \varepsilon^t \alpha_L{}^t \beta_P{}^t \gamma_{PLL}{}^t \delta_P{}^t \tau_P{}^t$.

Take the geometric mean of both sides of the above equations to get our preferred decomposition:

$p^t \cdot y^t / p^{t-1} \cdot y^{t-1} = \varepsilon^t \alpha^t \beta^t \gamma^t \delta^t \tau^t$.

Can re-organise to get:

$TFPG^t \equiv \{[p^t \cdot y^t / p^{t-1} \cdot y^{t-1}]/\alpha^t\}/\beta^t = \varepsilon^t \gamma^t \delta^t \tau^t$

A Nonparametric Approximation to the Cost Constrained Value Added Function

Assume that the production unit's period t production possibilities set S^t is the **conical free disposal hull of the period t actual production vector and past production vectors**. LP problem:

$$
\begin{aligned}
R^t(p,w,x) &\equiv \max_{\lambda} \{p\cdot(\Sigma_{s=1}^{t} y^s\lambda_s) ; w\cdot(\Sigma_{s=1}^{t} x^s\lambda_s) \le w\cdot x ; \lambda_1 \ge 0 ,\ldots, \lambda_t \ge 0\} \\
&= \max_{s} \{p\cdot y^s\, w\cdot x/w\cdot x^s : s = 1,2,\ldots,t\} \\
&= w\cdot x \max_{s} \{p\cdot y^s/w\cdot x^s : s = 1,2,\ldots,t\} \\
&= w\cdot x/\min_{s} \{w\cdot x^s/p\cdot y^s : s = 1,2,\ldots,t\} \\
&= w\cdot x/c^t(w,p)
\end{aligned}
$$

where $c^t(w,p)$ is the *period t nonparametric unit cost function*

TFP Growth for the U.S. Corporate Nonfinancial Sector, 1960-2014

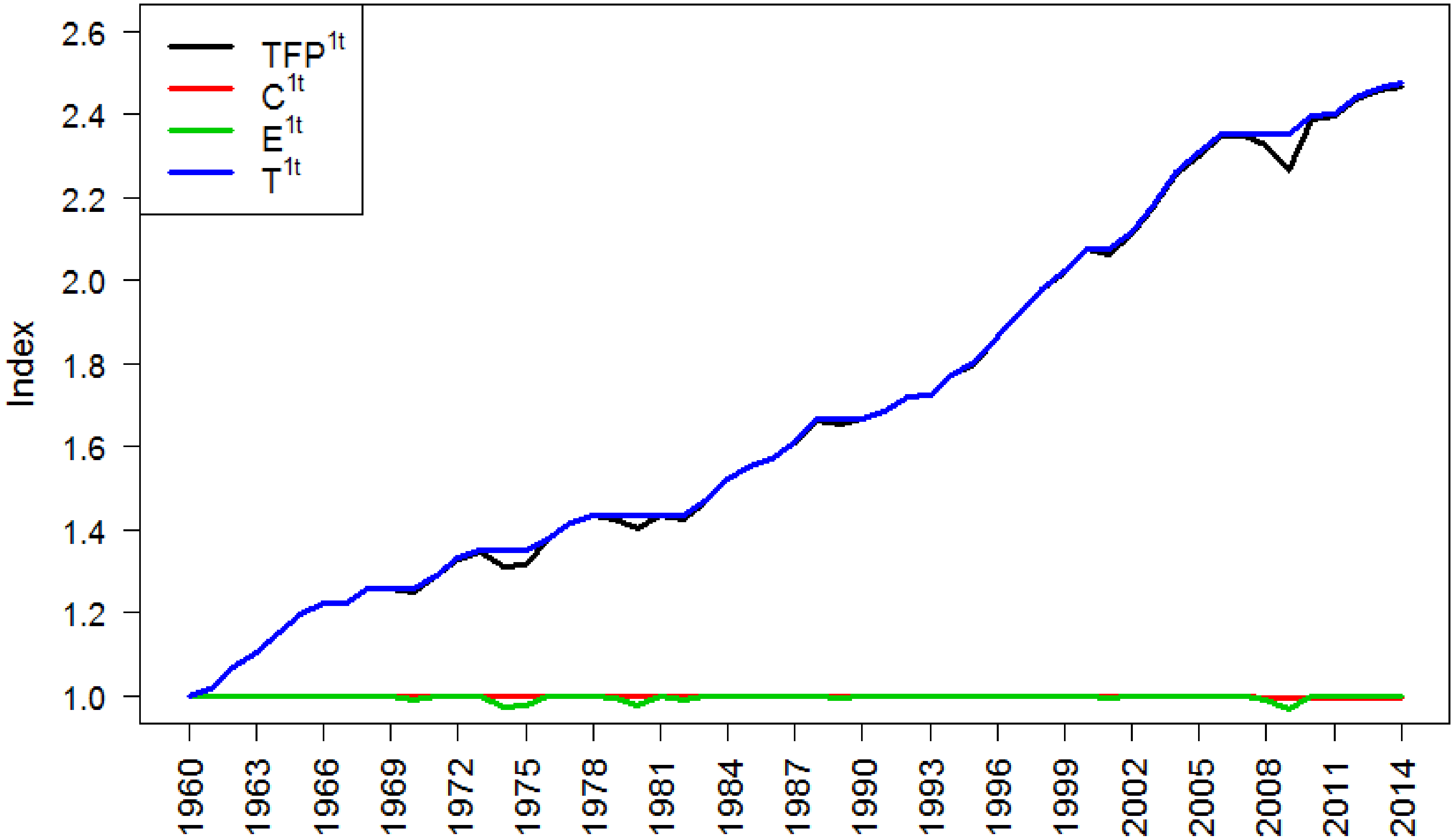
Use the (BEA, BLS, Fed Reserve) Integrated Macroeconomic Accounts to construct a data set for two major sectors of the U.S. economy in Diewert and Fox (2016) :

"Alternative User Costs, Rates of Return and TFP Growth Rates for the US Nonfinancial Corporate and Noncorporate Business Sectors: 1960-2014"

Sector 1: US Corporate Nonfinancial Sector

Sector 2: US Noncorporate Nonfinancial Sector

Figure 1: Sector 1 Level of TFP, Input Mix, Value Added Efficiency and Technology



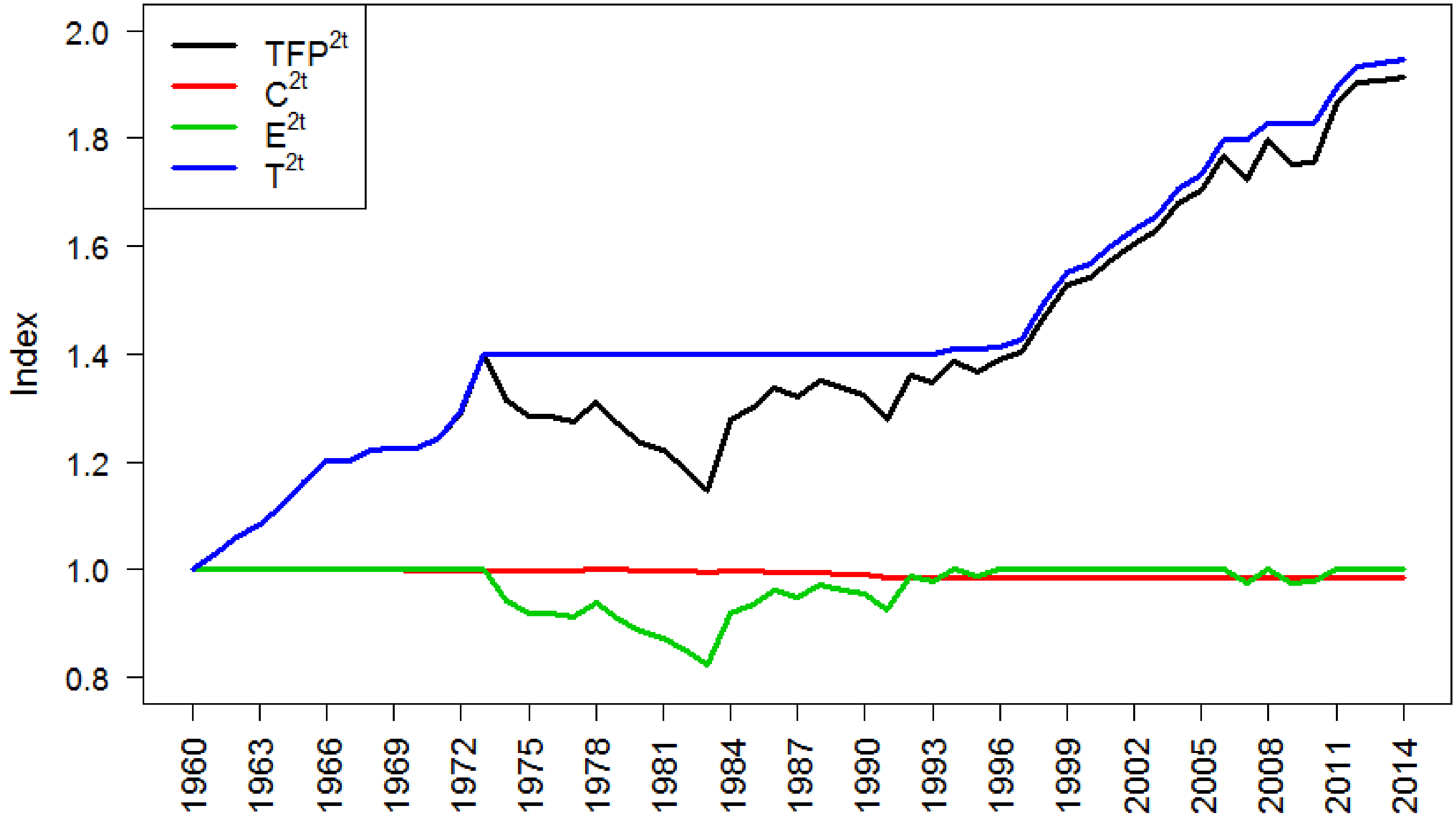
TFP Growth for the U.S. Corporate Nonfinancial Sector, 1960-2014

- There was a substantial decline in value added efficiency over the years 2006-2009

- TFP has grown at a slower than average rate since 2006. The level of TFP also fell in the 1974, 1979, 1982, 1989 and 2001 recessions when efficiency growth dipped below one.

- On the whole, TFP growth in the U.S. Corporate Nonfinancial Sector has been satisfactory.

Figure 2: Sector 2 Level of TFP, Input Mix, Value Added Efficiency and Technology



TFP Growth for the U.S. Noncorporate Nonfinancial Sector, 1960-2014

- The loss of value added efficiency in Sector 2 was massive over the 20 years 1974-1993. This loss of efficiency dragged down the level of Sector 2 TFP over these years. TFP growth resumed in 1994 and was excellent until 2006 when TFP growth again stalled with the exception of two good years of growth in 2011 and 2012.

- Illustrates the adverse influence of recessions when output falls but inputs cannot be adjusted optimally due to the fixity of many capital stock (and labour) components of aggregate input.

- Under these circumstances, production takes place in the interior of the production possibilities set and for Sector 2, the resulting waste of resources was substantial.

Summary

- Derived decompositions of nominal value added growth (and TFP growth) for a single sector into explanatory factors.
- In the paper, we also used two alternative approaches to relating the sectoral decompositions to a national growth decomposition:
  - a weighted average sectoral approach and
  - a national value added function approach.
- **The main advantage of our new approach is that our new nonparametric measure of technical progress never indicates technical regress**.
- During recessions, value added efficiency drops below unity and depresses TFP growth.

Summary

- For our U.S. data set, TFP growth is well explained as the product of value added efficiency growth times the rate of technical progress.
- For the U.S. Noncorporate Nonfinancial Sector, we found that the cost of recessions was particularly high.
- Implementation of the decompositions can provide key insights into the drivers of economic growth at a detailed sectoral level.
- Hence, we believe that they will provide new insights into the sources of economic growth.
- Our decompositions may also indicate data mismeasurement problems that can then be addressed by statistical agencies.

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