A Democratic Measure of Income Growth

Andrew Aitken and Martin Weale

16th May 2018
ESCoE Conference on Economic Measurement
London

National Accounts

- William Petty, FRS (1620-1687) - inventor of the term "Political arithmetic"- the study of the economic and demographic statistics of a state
- Produced the first national accounts
- James Meade (1907-1995) and Richard Stone (1913-1990)
- The first modern national accounts in 1941 (simultaneous with those in the Netherlands and Palestine)
- A focus on national income rather than output
- Now integrated with measurement of GDP

"That's your bloody GDP, not ours" – Newcastle Heckler

- It is well-known that GDP growth is not a good measure of growth in welfare.
  1. It is gross of depreciation. A welfare measure has to be net.
  2. Account needs to be taken of net income from abroad.
  3. Money net income should be deflated by the price of consumption, not the price of output.
  4. Some adjustment for population growth is needed.
  5. Real net national income growth *per capita* is a much better indicator of change in welfare.
- But aggregate or average real growth weights individual growth rates according to the *level* of income of each household.
- A plutocratic indicator of welfare growth. High earners count for more.
- A democratic measure takes the growth in the real income of each household and averages this across all households.

Democratic Growth

- Sig Prais (1959) developed a democratic price index. It calculates the change in prices based on the spending pattern of an average household. CPI uses total spending, so high spenders have more influence.
- Tony Atkinson (1971) developed "inequality-averse" measures of income.
- We take the geometric mean of household income (A special case of Atkinson's inequality aversion) and deflate using Prais democratic price index.
- The output is the growth rate in real household income averaged across all households.
- We discuss the role of this as an indicator of welfare accruing.

Atkinson Inequality Aversion

$$Y = \frac{\left(\sum_{i=1}^{n} y_i^{1-\rho}\right)^{\frac{1}{1-\rho}}}{n} \; (\rho \neq 1) \;\; \text{If } \rho = 1 \text{ then} \qquad Y = \sqrt[n]{\prod_{i=1}^{n} y_i}$$

- Assume now that there is an aggregate price measure P and quantity measure (real income) Q so that Y=PQ. Each household has a price index p_i and a quantity index q_i with $y_i = p_i\, q_i$ so that

$$PQ = \frac{\left(\sum_{i=1}^{n} p_i^{1-\rho} q_i^{1-\rho}\right)^{\frac{1}{1-\rho}}}{n}$$

- Now take logs

$$logY = logP + logQ = \frac{log\left(\sum_{i=1}^{n} p_i^{1-\rho} q_i^{1-\rho}\right)}{1-\rho} - logn$$

- And differentiate

$$\frac{\Delta Y}{Y} = \frac{\Delta P}{P} + \frac{\Delta Q}{Q} = \frac{\sum_{i=1}^{n} \Delta p_i p_i^{-\rho} q_i^{1-\rho} + \Delta q_i p_i^{1-\rho} q_i^{-\rho}}{\left(\sum_{i=1}^{n} p_i^{1-\rho} q_i^{1-\rho}\right)}$$

A General Measure

$$\frac{\Delta P}{P}+\frac{\Delta Q}{Q}=\frac{\sum_{i=1}^{n}(\frac{\Delta p_i}{p_i})p_i^{1-\rho}q_i^{1-\rho}+(\frac{\Delta q_i}{q_i})p_i^{1-\rho}q_i^{1-\rho}}{\left(\sum_{i=1}^{n}p_i^{1-\rho}q_i^{1-\rho}\right)}=\frac{\sum_{i=1}^{n}\left(\frac{\Delta p_i}{p_i}\right)y_i^{1-\rho}+(\frac{\Delta q_i}{q_i})y_i^{1-\rho}}{\left(\sum_{i=1}^{n}y_i^{1-\rho}\right)}$$

The household weights are $y_i^{1-\rho}$
If ρ=1 all households are given equal weight
If $\rho > 1$ then poor households are given more weight than rich households.
In the special case with $\rho = 1$

$$\frac{\Delta Y}{Y}=\frac{\Delta P}{P}+\frac{\Delta Q}{Q}=\frac{\sum_{i=1}^{n}\Delta p_i/p_i+\Delta q_i/q_i}{n}$$

$\frac{\Delta P}{P}=\frac{\sum_{i=1}^{n}\Delta p_i/p_i}{n}$ is the Prais Index

and we can derive $\frac{\Delta Q}{Q}=\frac{\Delta Y}{Y}-\frac{\Delta P}{P}$ to get the democratic growth rate.

Towards a Welfare Interpretation

- If saving is the only source of income growth, the rate of change of real income is the real rate of return on saving
- $\dot{q}_{it} = r_{it}(q_{it} - \frac{\pi'_t c_{it}}{p_{it}})$
- Gives a relationship between current income and current and future consumption
- $q_{it} = \int_t^{\infty} (r_{i\tau}\pi'_\tau c_{i\tau} e^{-\int_t^{\tau} r_{i\vartheta} d\vartheta} / p_{i\tau}) d\tau$
- and , with inter-temporally efficient allocation, a welfare interpretation of saving

$$\frac{d}{dt}\int_t^{\infty} u_i(c_{i\tau})e^{-\theta\tau}d\tau = p_{it}\frac{\partial z_{it}}{\partial x_{it}}\left(\int_t^{\infty}(r_{i\tau}\pi'_\tau c_{i\tau}e^{-\int_t^{\tau} r_{i\vartheta}d\vartheta}/p_{i\tau})d\tau - \pi'_t c_{it}/p_{it}\right)$$

- The rate of change of life-time utility is saving multiplied by the marginal utility of money, (Sefton and Weale 2006)
- Requires individual homotheticity (constant expenditure shares)
- Otherwise there is no single measure of real income even for an individual household and the Divisia measure is path-dependent

How Limiting is the Divisia Assumption?

- Redding and Weinstein(CEP Paper 2016) argue that demand functions are homothetic but prone to preference shocks. Consider a CES utility function

$$u_i = \sum \left(\frac{c_{ij}}{\delta_{ij}}\right)^{1-\sigma}$$

- The price index is $$p_i^{\frac{1-\sigma}{\sigma}} = \sum_j (\pi_j \delta_{ij})^{\frac{\sigma-1}{\sigma}}$$

- And the expenditure shares are $$\omega_{ij} = \frac{(\pi_j \delta_{ij})^{\frac{\sigma-1}{\sigma}}}{\sum_j (\pi_j \delta_{ij})^{\frac{\sigma-1}{\sigma}}}$$

- Differentiating $$\frac{dp_i}{p_i} p_i^{1-\sigma} = \sum_j \frac{d\pi_j}{\pi_j} (\pi_j \delta_{ij})^{1-\sigma} + \sum_j \frac{d\delta_j}{\delta_j} (\pi_j \delta_{ij})^{1-\sigma}$$

- So $$\frac{dp_i}{p_i} = \sum_j \frac{d\pi_j}{\pi_j} \omega_{ij} + \sum_j \frac{d\delta_j}{\delta_j} \omega_{ij}$$

- And Divisia is valid if $$\sum_j \frac{d\delta_j}{\delta_j} \omega_{ij} = 0$$

- Using scanner data Redding and Weinstein suggest this condition is met

Aggregating with Unit Inequality Aversion

- Suppose we also have an exogenous source of growth, h_{it} so that

$$\dot{q}_{it} = r_{it}\left(q_{it} - \frac{\pi'_t c_{it}}{p_{it}}\right) + h_{it}$$

and

$$\frac{\dot{q}_{it}}{q_{it}} = \frac{r_{it}}{y_{it}\partial z_{it}/\partial x_{it}} \frac{d}{dt}\int_t^\infty u_i(c_{i\tau})e^{-\theta\tau}d\tau \; + \frac{h_{it} - r_{it}\int_t^\infty h_{i\tau}e^{-\int_t^\tau r_{i\vartheta}d\vartheta}d\tau}{q_{it}}$$

Set $s_{it} = \frac{h_{it} - r_{it}\int_t^\infty h_{i\tau}e^{-\int_t^\tau r_{i\vartheta}d\vartheta}d\tau}{q_{it}}$

Then

$$\frac{\dot{Q}_t(1)}{Q_t(1)} = \left(\sum_{i=1}^{N}\left\{\frac{r_{it}}{y_{it}\frac{\partial z_{it}}{\partial x_{it}}}\frac{d}{dt}\int_t^\infty u_i(c_{i\tau})e^{-\theta\tau}d\tau + s_{it}\right\}\right)/N$$

Aggregate Income and a Market Social Welfare Function

- The growth rate of the aggregate (plutocratic) price index, used to calculate real income is the average of the growth rate of the individual price indices, weighted by the consumption, not the income, of each household.
- Aggregate utility is, with α_i market weights, defined as
- $U = \sum_i \alpha_i \int_t^\infty u_i\,(c_{i\tau})e^{-\theta\tau}d\tau$
- $Z^M\,(\pi_i, X_t)$ is indirect utility with resources efficiently allocated, i.e. if
- $\frac{\partial Z^M(\pi_t, X_t)}{\partial X_t} = \alpha_i \frac{\partial z_i(\pi_t, x_{it})}{\partial x_{it}}$
- Households with high marginal utility of income have low weights
- The price index in the national accounts is computed using consumption weights rather than income weights. Then

$$\frac{\dot{Q}^C{}_t(0)}{Q^C{}_t(0)} = \frac{r_t}{Y_t \partial Z^M/\partial X_t} \frac{d}{dt} \sum_i \alpha_i u_i\,(c_{i\tau})e^{-\theta\tau}d\tau \; + \frac{H_t - r_t \int_t^\infty H_\tau e^{-\int_t^\tau r_\vartheta d\vartheta} d\tau}{Q^C{}_t(0)}$$

The Price Index as a Constant-scaling Cost of Living Index

- Write indirect utility as a function of prices, household money consumption and the elasticity of substitution

- $z_i(\pi_t, x_{it}, \sigma) = \frac{1}{1-\sigma}\left(\frac{x_{it}}{p_i(\pi_t)}\right)^{1-\sigma} \qquad (\sigma \neq 1)$

$$= log\left(\frac{x_{it}}{p_i(\pi_t)}\right) \qquad (\sigma = 1)$$

- A utilitarian welfare function

$$Z^U(\pi_t, x_{1t}, \dots x_{Nt}, \sigma) = \sum_i \frac{1}{N(1-\sigma)}\left(\frac{x_{it}}{p_i(\pi_t)}\right)^{1-\sigma}$$

The Price Index as a Constant-scaling Cost of Living Index

- Introduce a common scaling factor, μ_t
- $Z^U(\pi_t, x_{1t}, \dots x_{Nt}, \sigma) = \sum_i^N \frac{1}{N(1-\sigma)} \left(\frac{\mu_t x_{it}}{p_i(\pi_t)}\right)^{1-\sigma}$
- $\dot{\mu}_t \frac{\partial Z_U}{d\mu_t} + \sum_j \frac{\partial Z^U}{\partial \pi_{jt}} \dot{\pi}_{jt} = 0$
- It shows how much incomes need to change so that social welfare is unaffected by price changes.
- $\frac{\dot{\mu}_t}{\mu_t} = \frac{\sum_i \left(\frac{x_{it}}{p_i(\pi_t)}\right)^{1-\sigma} \frac{\dot{p}_i(\pi_t)}{p_i(\pi_t)}}{\sum_i \left(\frac{x_{it}}{p_i(\pi_t)}\right)^{1-\sigma}}$
- The rate of growth of the scaling factor is the growth rate of each household's price index weighted by real consumption to the power 1-σ. If σ=1, we have the democratic price index.

Income, Consumption and Welfare

- The utility accruing from income is the utility accruing from consumption plus saving multiplied by the marginal utility of consumption.

$$z^y{}_i(\pi_t, x_{it}, y_{it}) = z_i(\pi_i, x_{it}) + (y_{it} - x_{it})\frac{\partial z_i(\pi_t, x_{it})}{\partial x_{it}}$$

- Consider a first-order Taylor approximation

$$z^y{}_i(\pi_t, x_{it}, y_{it}) \cong z_i(\pi_i, y_{it})$$

- Income is an indicator of welfare accruing

$$\frac{\dot{P}^Y{}_t}{P^Y{}_t} = \frac{\sum_i \left(\frac{y_{it}}{p_i(\pi_t)}\right)^{1-\sigma} \frac{\dot{p}_i(\pi_t)}{p_i(\pi_t)}}{\sum_i \left(\frac{y_{it}}{p_i(\pi_t)}\right)^{1-\sigma}}$$

- It then follows that

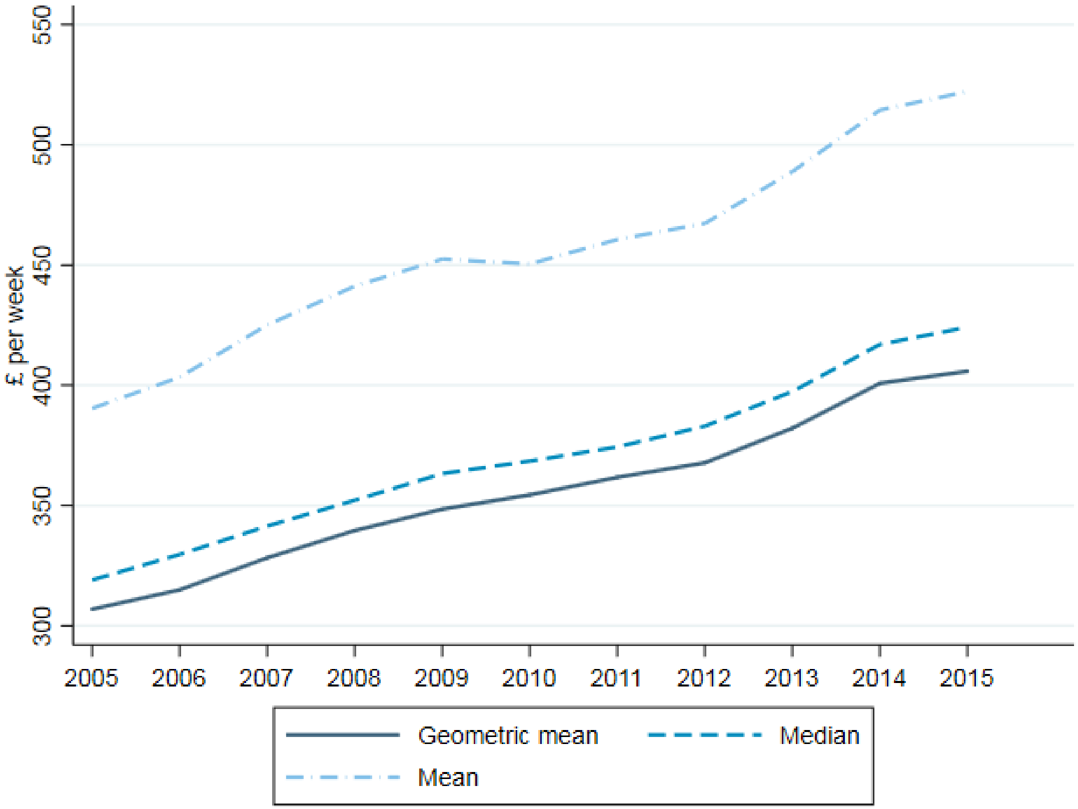
$$\frac{\dot{Q}^Y{}_t}{Q^Y{}_t} = \frac{\sum_i \frac{\dot{q}_i(\pi_t)}{q_i(\pi_t)}(q_{it})^{1-\sigma}}{\sum_i (q_{it})^{1-\sigma}}$$

- The weights are based on real rather than nominal incomes, but with σ=1 this is the democratic growth rate.

Application

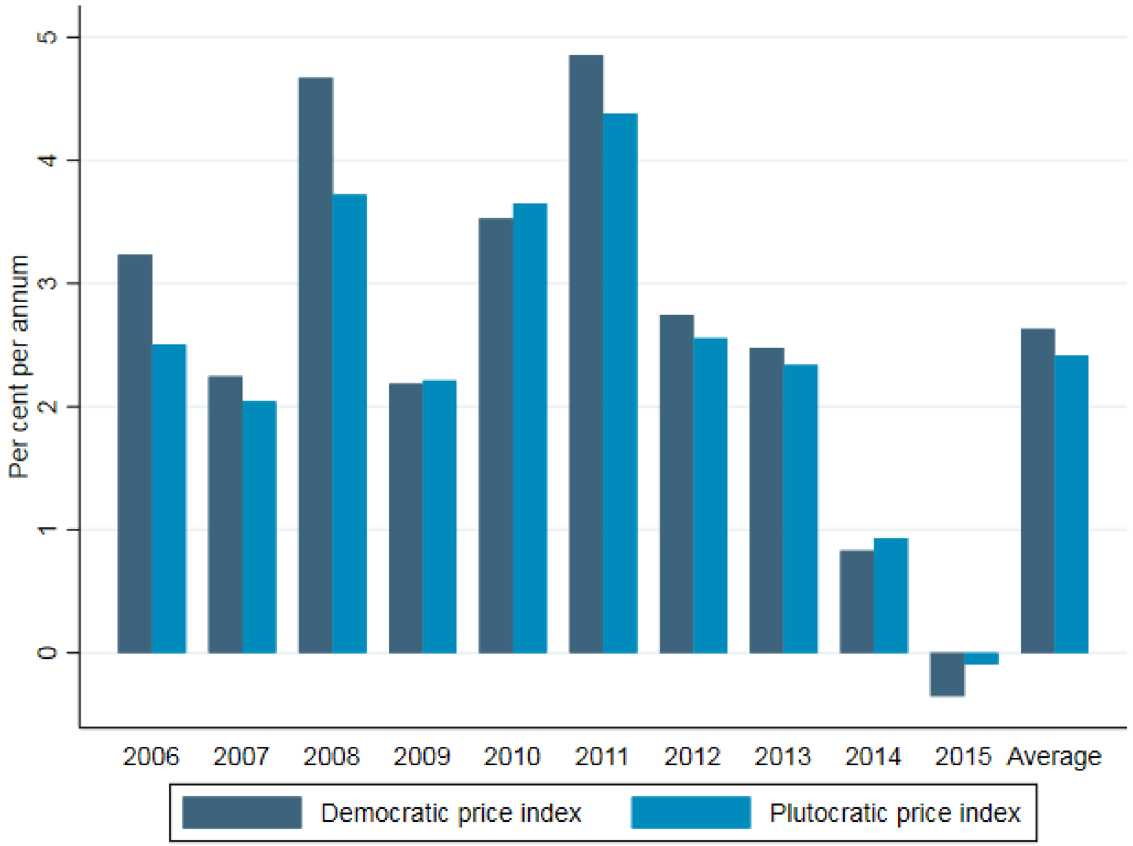
- Adjust household income for household size
- Use democratic CPI produced by Tanya Flower and Philip Wales at ONS
- Results for household disposable income after housing costs taken from Households Below Average Income dataset

Three Measures of Central Tendency of Nominal Household Income after Housing Costs

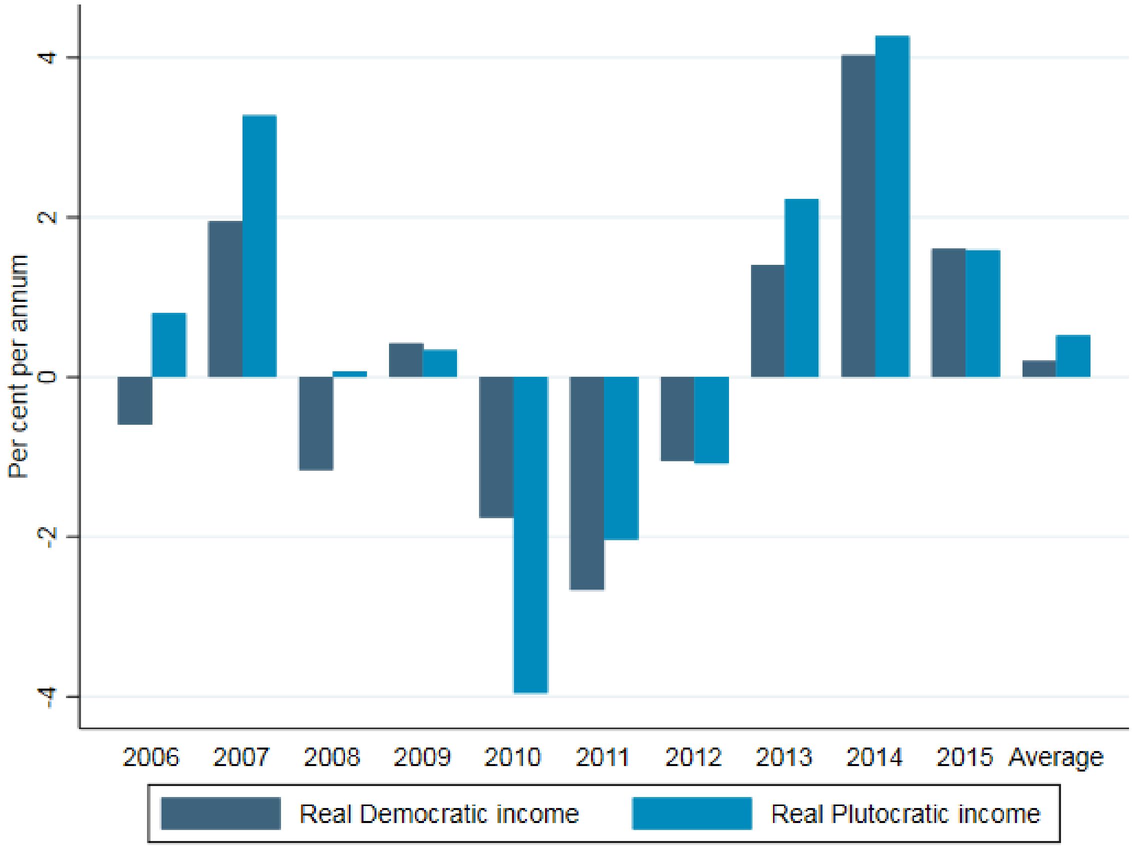


Data Source: Households below Average Income

Growth Rates of Democratic and Plutocratic Price Indices



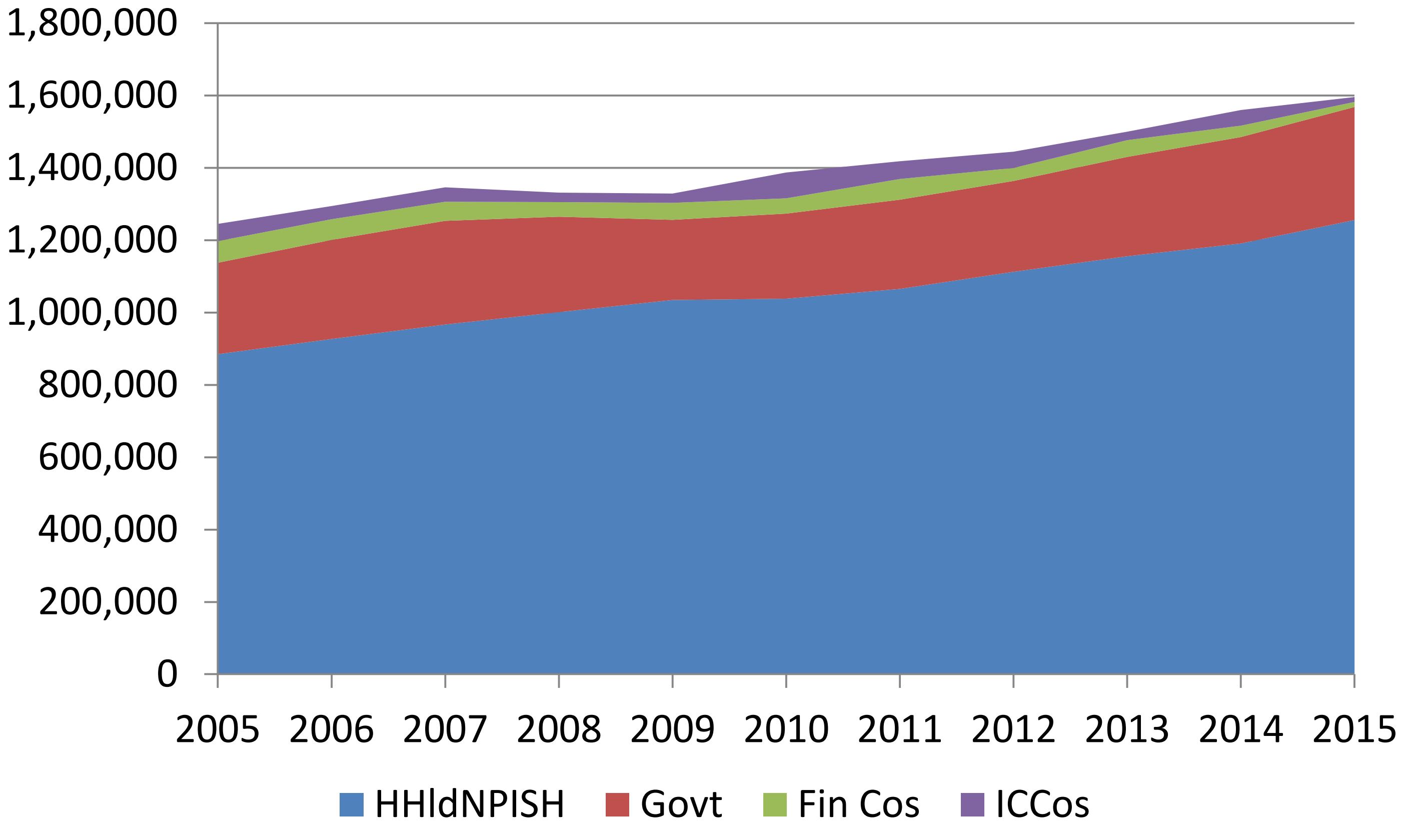
Growth Rates of Real Democratic and Plutocratic Income



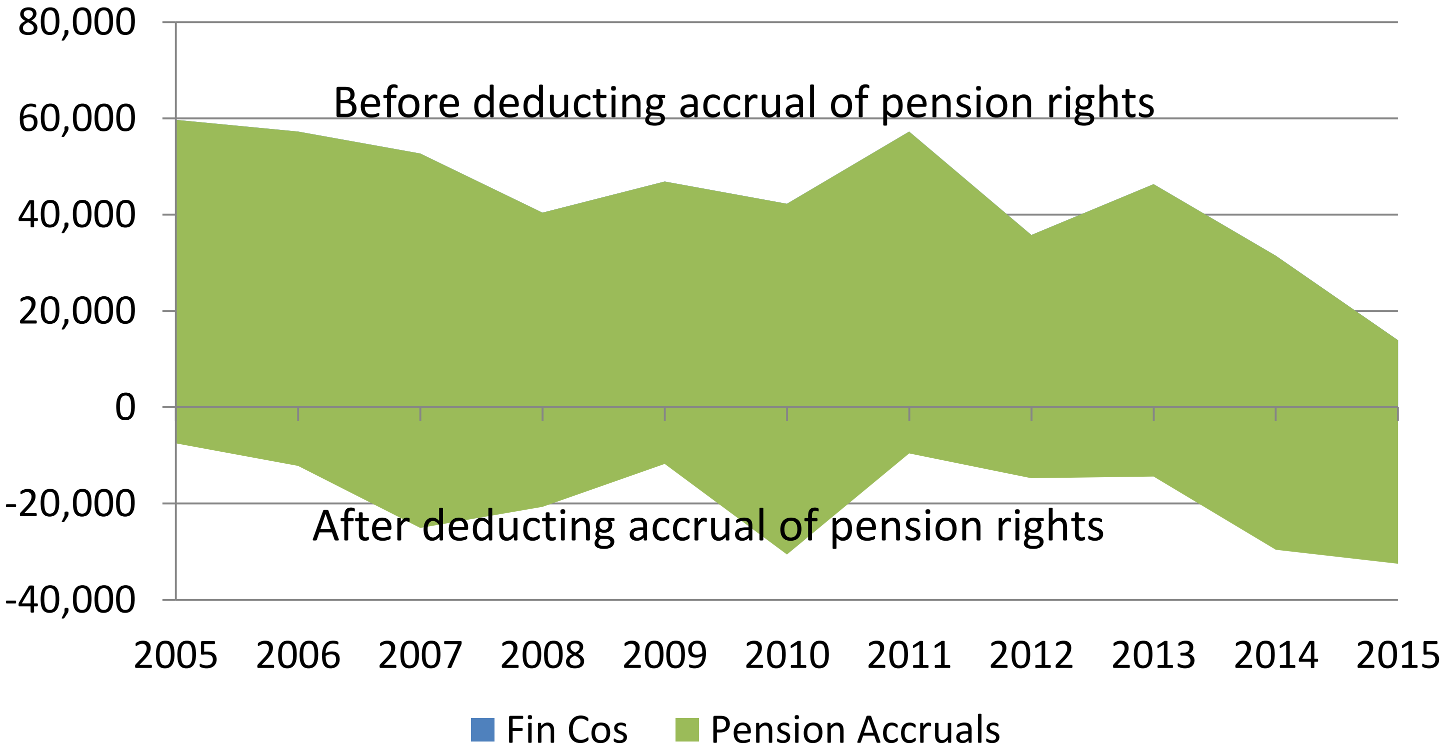
Application to National Income

- Results so far relate to household income.
- National income includes components not allocated to households
    1. Undistributed income of corporations
    2. Tax revenues spent on government-provided consumption and investment
    3. Income accruing to pension funds
- Also major doubts about the accuracy of survey data

Allocation of Net National Disposable Income (£m)



Pension Rights and Financial Companies (£m)



This graph shows the disposable income (saving) of financial companies before and after adjusting for accruals of pension rights

Allocating the whole of National Income to Households to deliver a Democratic Measure of National Income Growth

£bn 2015	Initial Income	Pensions	Dividends	Govt cons	Govt saving	Reallocated Income
UK total economy	1,591,432					1,591,432
Non-financial corporations	19,729		-19,729			0
Financial corporations	13,924	-46,546	32,622			0
General government	309,695			-362,062	52,367	0
Households and NPISH	1,248,084	46,546	-12,893	362,062	-52,367	1,591,432

The Scale of Misreporting, 2013

Component	National Accounts Total	Microsource Total	Coverage Rate (%)
Macro resources (received):			
Operating surplus	130,150	68,060	52
Mixed income	110,469	63,274	57
Wages and salaries	711,054	663,206	93
Net property income received	149,811	34,396	23
Social benefits other than STiK	332,504	231,013	69
Social transfers in kind	273,509	179,603	66
A Total	**1,707,497**	**1,239,552**	**73**
Macro uses (paid):			
Current taxes on income and wealth	195,524	142,923	73
Employers actual social contributions	136,091	59,606	44
Households social contributions	67,528	62,945	93
B Total	**399,143**	**265,474**	**67**
Household Gross Disposable Income (A-B)	**1,308,354**	**974,078**	**74**
Memo: Gross Prop. Inc. excl. Rent	75,903	21,651	29
Source: Office for National Statistics and own calculations			

Imputation Issues and Approaches

- Scaling widely used (e.g. in ONS work on consumption)
- Scaling preserves zeroes
- Scaling will not work for sources of income omitted from LCFS - e.g. undistributed accruals to pension funds.
- We found a higher proportion of zeros in LCFS than in other sources (e.g. SPI and HBAI)
- Need to model both the probability of a non-zero receipt and the magnitude of the receipt conditional on being non-zero
- In contrast to scaling, this has to be stochastic - there is not going to be any covariate which exactly identifies non-zero recipients in HBAI or SPI

Categorical Imputation using Ordered Probit Models I

- We adopt a flexible approach structured round an ordered probit model
- We convert the data in our source datasets (SPI for interest and dividend income/WAS for pensions) into a large number of categories and fit ordered probit models to these
- Covariates have to be variables available both in the source surveys and in LCFS
- Simulating these models provides stochastic categorical estimates which can be imputed into LCFS

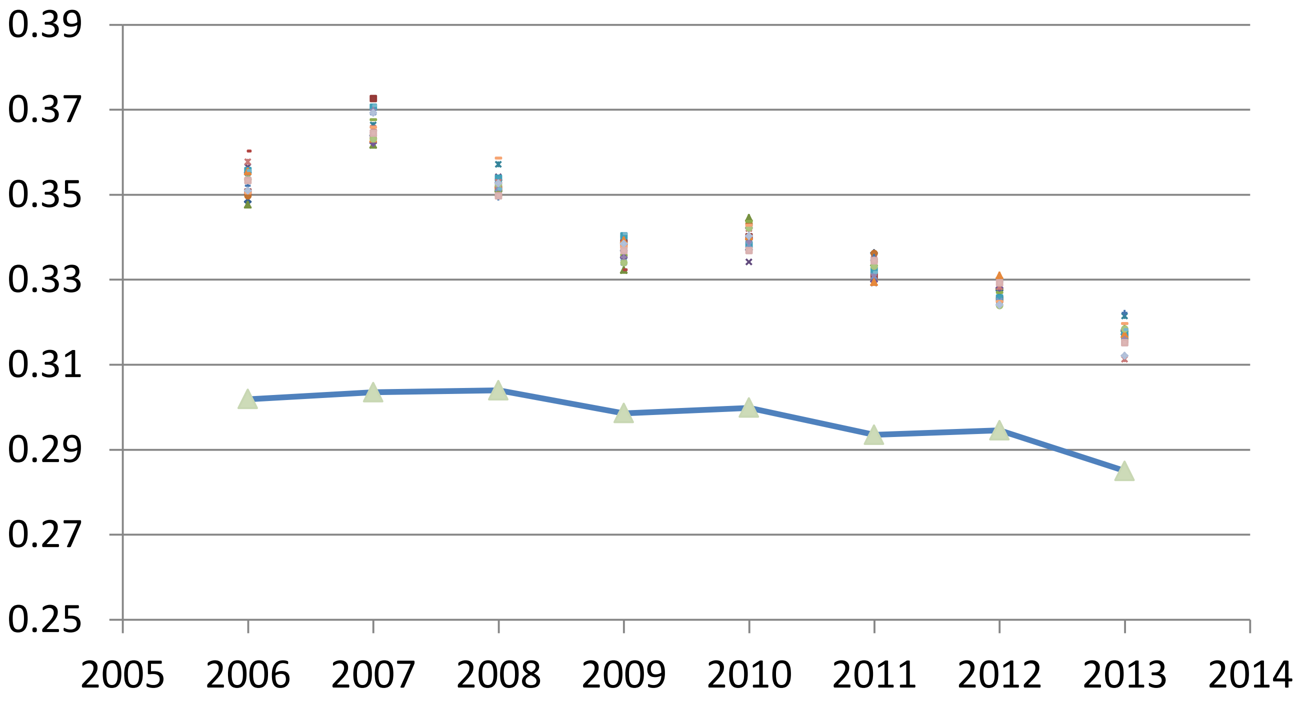
Categorical Imputation using Ordered Probit Models II

- Reconciliation of survey data with the macro data requires appropriate handling of the upper tail
- Use a Pareto type-1 distribution for observations in the upper tail of the distribution
- Need to take into account correlation between random components of imputed variables
- Estimate a correlation matrix using WAS (which does allow joint estimation but is not the ideal source) for the random components

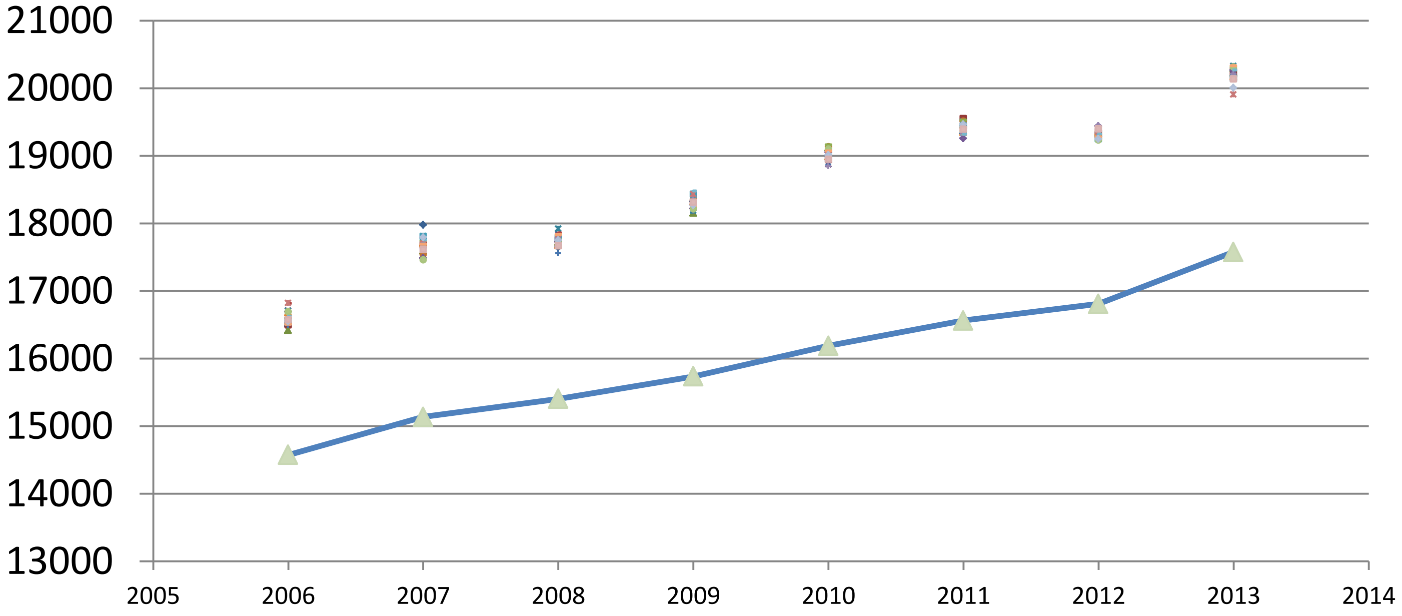
Simulations

- Examine the effect of including imputed pension and investment income on measures of inequality such as Gini and geometric mean of income
- Present results from 20 simulations
  – Preliminary due to top-coding of labour income in LCFS data

Estimates of the Gini Coefficient: with pension and investment income imputations (20 draws)



Estimates of the Geometric Mean of Household Income: with pension and investment income imputations (20 draws)



Future Work

- Currently working on full version of LCFS data, rather than top-coded data
- Extend existing democratic price indices to cover public consumption (drawing heavily on ONS work on Earnings, Taxes and Benefits dataset)
- Produce a democratic measure of real disposable national income in the Autumn.