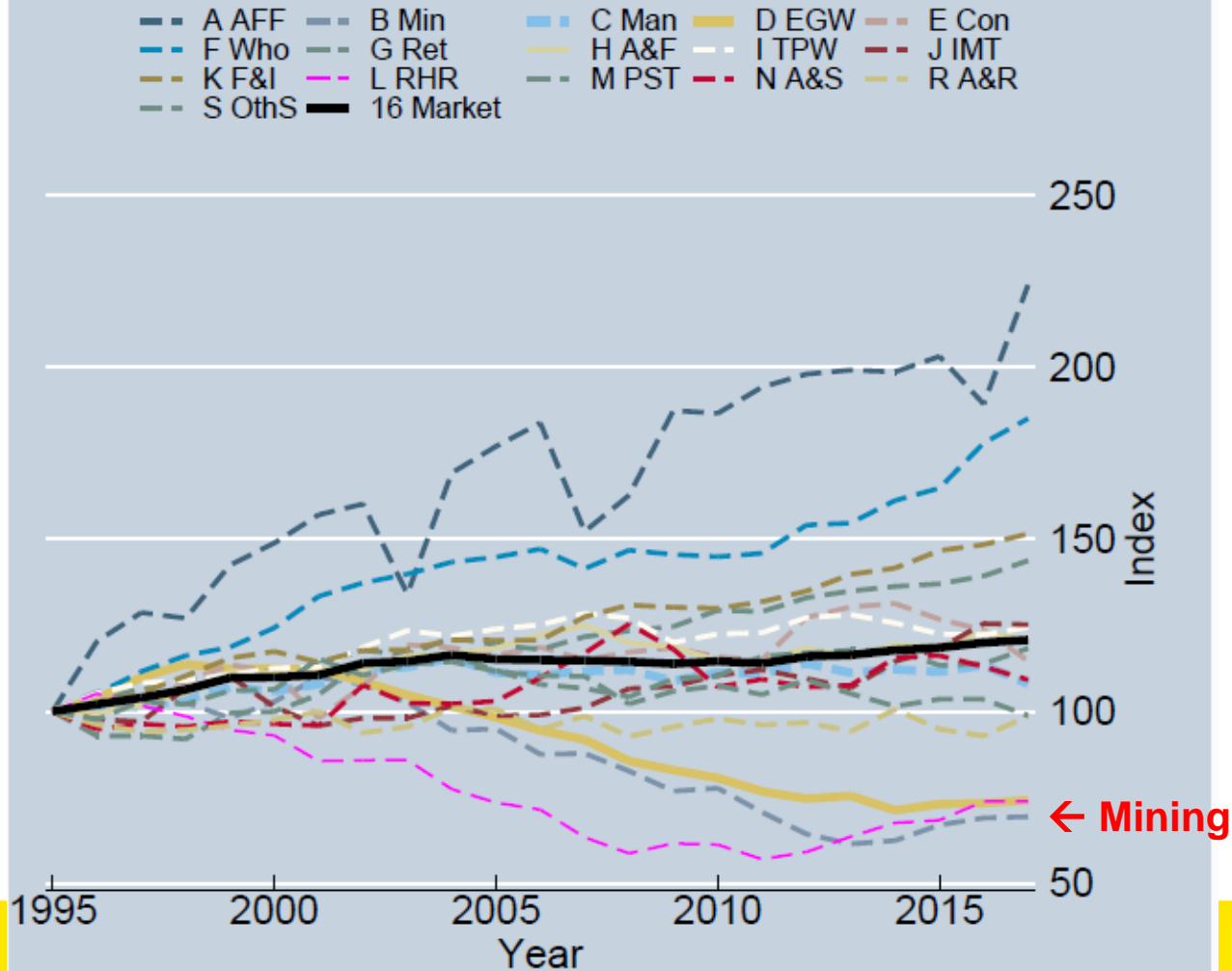


# Value added and Productivity Decompositions with Natural Capital

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# ABS MFP:



## Summary

- **Decompose nominal value added growth for the Australian Mining Sector, taking into account natural capital, specifically, **subsoil assets**.**
- **Explanatory factors are**
  - **efficiency changes,**
  - **changes in output prices,**
  - **changes in primary inputs,**
  - **changes in input prices, and**
  - **technical progress**

## Summary

- **Build on the work of Brandt, Schreyer and Zipperer (2017), Diewert and Fox (2016), Diewert and Fox (2018) and Zeng, Diewert and Fox (2018) and Hoang (2018).**
- **Use access to privileged data from the Australian Bureau of Statistics on subsoil assets.**
- **Consider impact of alternative valuation methods in deriving the capital services from such natural capital.**
- **Results provide insights into the drivers of value added and productivity growth.**

## Value Added Decomposition

- **Need sector's best practice technology for the periods under consideration.**
- **Could use econometric or nonparametric (DEA) techniques**
- **Use a Free Disposal Hull approach – no convexity assumptions**
- **Our approach has the advantage that it does not involve econometric estimation, and involves only observable data.**
- **Simple enough to be implemented by national statistical offices**

## Value Added Decomposition

- A sector produces  $M$  net outputs,  $y \equiv [y_1, \dots, y_M]$ , using  $N$  primary inputs  $x \equiv [x_1, \dots, x_N] \geq 0_N$ .
- If  $y_m > 0$ , then the sector produces the  $m^{\text{th}}$  net output during period  $t$  while if  $y_m < 0$ , then the sector uses the  $m^{\text{th}}$  net output as an intermediate input.
- Strictly positive vector of net output prices  $p \equiv [p_1, \dots, p_M] \gg 0_M$  and strictly positive vector of input prices  $w \equiv [w_1, \dots, w_N] \gg 0_N$
- *Period  $t$  production possibilities set* for the sector  $S^t$  satisfies some (minimal) regularity conditions

## Value Added Decomposition

*Period  $t$  cost constrained value added function:*

$$R^t(p,w,x) \equiv \max_{y,z} \{p \cdot y : (y,z) \in S^t; w \cdot z \leq w \cdot x\}$$

If  $S^t$  is a cone, so that production is subject to constant returns to scale, Diewert-Fox show that

$$R^t(p,w,x) \equiv w \cdot x / c^t(w,p)$$

where  $c^t(w,p)$  is the unit cost function for producing a unit of value added.

## Value Added Decomposition

Observed value added,  $p^t \cdot y^t$ , may not equal the optimal value added.

**Value added efficiency** of the sector during period  $t$ :

$$e^t \equiv p^t \cdot y^t / R^t(p^t, w^t, x^t) \leq 1$$

**Change in value added efficiency:**

$$\varepsilon^t \equiv e^t / e^{t-1}$$

If  $\varepsilon^t > 1$ , value added efficiency has *improved*, *fallen* if  $\varepsilon^t < 1$ .



## Value Added Decomposition

Family of *input quantity indexes*:

$$\beta(x^{t-1}, x^t, w) \equiv w \cdot x^t / w \cdot x^{t-1}.$$

Laspeyres and Paasche type special cases:

$$\beta_L^t \equiv w^{t-1} \cdot x^t / w^{t-1} \cdot x^{t-1} ;$$

$$\beta_P^t \equiv w^t \cdot x^t / w^t \cdot x^{t-1} .$$

Preferred overall measure of input quantity growth is the geometric average of the above two estimates of input growth (Fisher index):

$$\beta^t \equiv [\beta_L^t \beta_P^t]^{1/2}.$$

## Value Added Decomposition

Family of **output price indexes**:

$$\alpha(p^{t-1}, p^t, w, x, s) \equiv R^s(p^t, w, x) / R^s(p^{t-1}, w, x).$$

Laspeyres and Paasche type special cases:

$$\alpha_L^t \equiv R^{t-1}(p^t, w^{t-1}, x^{t-1}) / R^{t-1}(p^{t-1}, w^{t-1}, x^{t-1}) ;$$

$$\alpha_P^t \equiv R^t(p^t, w^t, x^t) / R^t(p^{t-1}, w^t, x^t).$$

Preferred overall measure of output price growth:

$$\alpha^t \equiv [\alpha_L^t \alpha_P^t]^{1/2}$$

## Value Added Decomposition

Family of *input mix indexes*:

$$\gamma(w^{t-1}, w^t, p, x, s) \equiv R^s(p, w^t, x) / R^s(p, w^{t-1}, x)$$

Laspeyres and Paasche type special cases:

$$\gamma_{LPP}^t \equiv R^t(p^{t-1}, w^t, x^t) / R^t(p^{t-1}, w^{t-1}, x^t);$$

$$\gamma_{PLL}^t \equiv R^{t-1}(p^t, w^t, x^{t-1}) / R^{t-1}(p^t, w^{t-1}, x^{t-1}).$$

Take the geometric average of the above two measures:

$$\gamma^t \equiv [\gamma_{LPP}^t \gamma_{PLL}^t]^{1/2}.$$

## Value Added Decomposition

Family of *technical progress indexes*:

$$\tau(t-1, t, p, w, x) \equiv R^t(p, w, x) / R^{t-1}(p, w, x)$$

$$\tau_L^t \equiv R^t(p^{t-1}, w^{t-1}, x^t) / R^{t-1}(p^{t-1}, w^{t-1}, x^t).$$

$$\tau_P^t \equiv R^t(p^t, w^t, x^{t-1}) / R^{t-1}(p^t, w^t, x^{t-1}).$$

For the cone case:

$$\tau_L^t = c^{t-1}(w^{t-1}, p^{t-1}) / c^t(w^{t-1}, p^{t-1}) \text{ and } \tau_P^t = c^{t-1}(w^t, p^t) / c^t(w^t, p^t)$$

(independent of  $x$ )

$$\tau^t \equiv [\tau_L^t \tau_P^t]^{1/2}.$$

## Value Added Decomposition

With CRS, **five factors** that explain value added growth:

1. Change in cost constrained value added efficiency:  $\varepsilon^t \equiv e^t/e^{t-1}$
2. Change in output prices:  $\alpha(p^{t-1}, p^t, w, x, s)$
3. Change in input quantities:  $\beta(x^{t-1}, x^t, w)$
4. Change in input prices:  $\gamma(w^{t-1}, w^t, p, x, s)$
5. Changes due to technical progress:  $\tau(t-1, t, p, w, x)$

**Value Added Growth:**  $p^t \cdot y^t / p^{t-1} \cdot y^{t-1} = \varepsilon^t \alpha^t \beta^t \gamma^t \delta^t \tau^t$

**TFP Growth:**  $\{[p^t \cdot y^t / p^{t-1} \cdot y^{t-1}] / \alpha^t\} / \beta^t = \varepsilon^t \gamma^t \delta^t \tau^t$

## Value Added Decomposition

Approximate the production unit's period  $t$  production possibilities set  $S^t$  by the conical free disposal hull of the period  $t$  actual production vector and past production vectors.

$R^t(p,w,x)$  can be estimated by solving a very simple one constraint linear programming problem:

$$R^t(p,w,x) \equiv \max_{\lambda} \{p \cdot (\sum_{s=1}^t y^s \lambda_s) ; w \cdot (\sum_{s=1}^t x^s \lambda_s) \leq w \cdot x ; \lambda_1 \geq 0, \dots, \lambda_t \geq 0\}$$
$$= w \cdot x / c^t(w,p)$$

where  $c^t(w,p)$  is the *period  $t$  nonparametric unit cost function that corresponds to  $R^t(p,w,x)$ .*

# Valuation of Natural Capital: Resource Rent Method

World Bank; Brandt, Schreyer & Zipperer 2017:

Unit rent:

$$u = p \cdot \alpha - w \cdot \beta$$

$\alpha$  = vector of final product amounts generated by 1 unit of resource

$p$  = the corresponding market output price vector

$\beta$  = a positive vector of input requirements for mining one unit

$w$  = the corresponding market input price vector.

## Valuation of Natural Capital: **Residual Method**

Used by Statistics Canada, one of the approaches recommended by the UN SEEA, when the resource rent can't be directly calculated.

The resource rent is then:

**Gross Operating Surplus less user cost of produced capital**

**Split GOS into returns on produced capital and returns on natural capital.**

Effectively an indirect measure of resource rent.



## Valuation of Natural Capital: **Traditional User Cost**

Similar to the approach used by NSOs in calculating capital services:

$$P^0[r - i + (1+i)\delta]$$

$P^0$  = beginning of the period price for a unit of the resource

$r$  = one period opportunity cost of capital

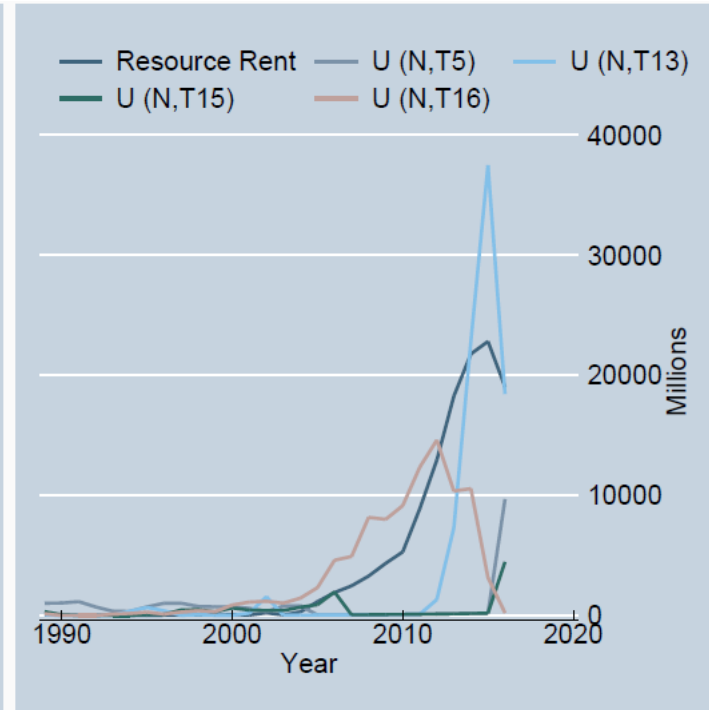
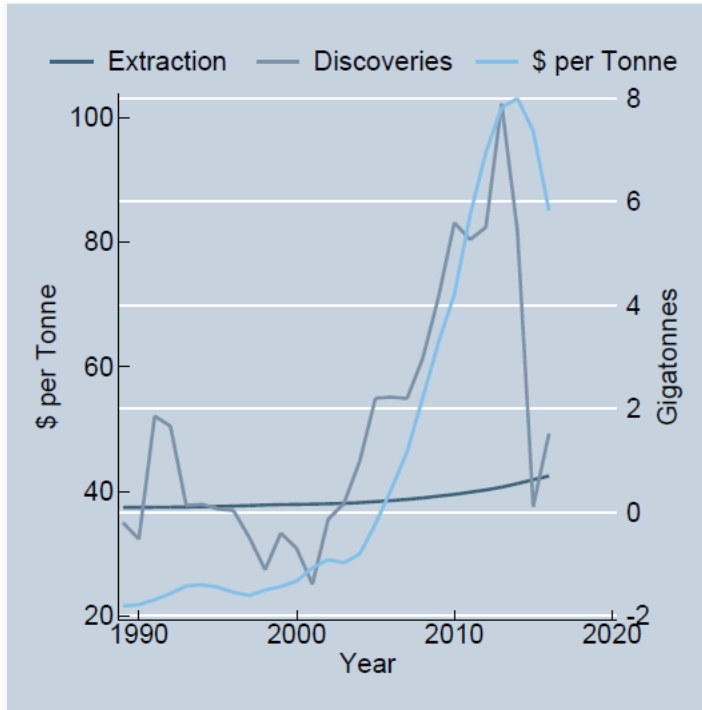
$i$  = inflation rate for a unit of the resource

$\delta$  = depletion rate (rather than the usual depreciation rate)

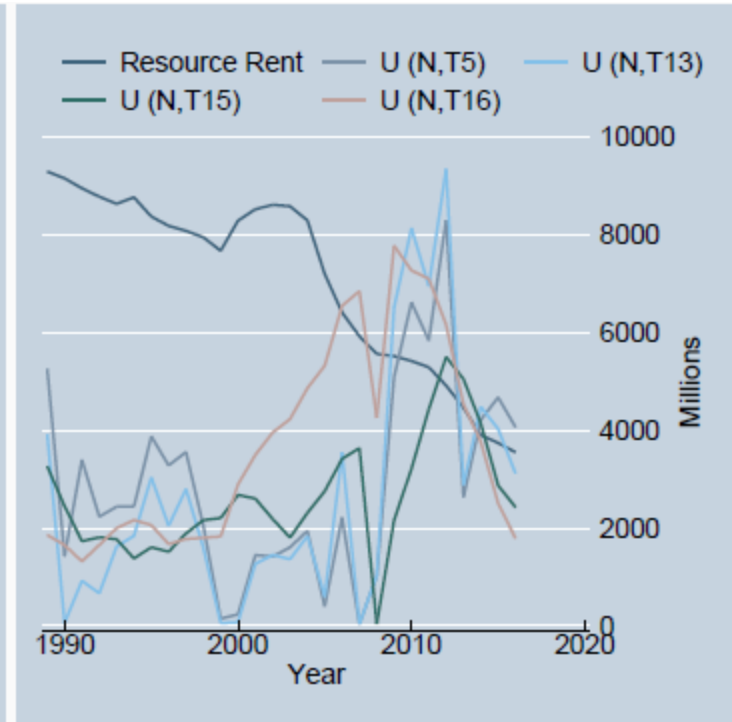
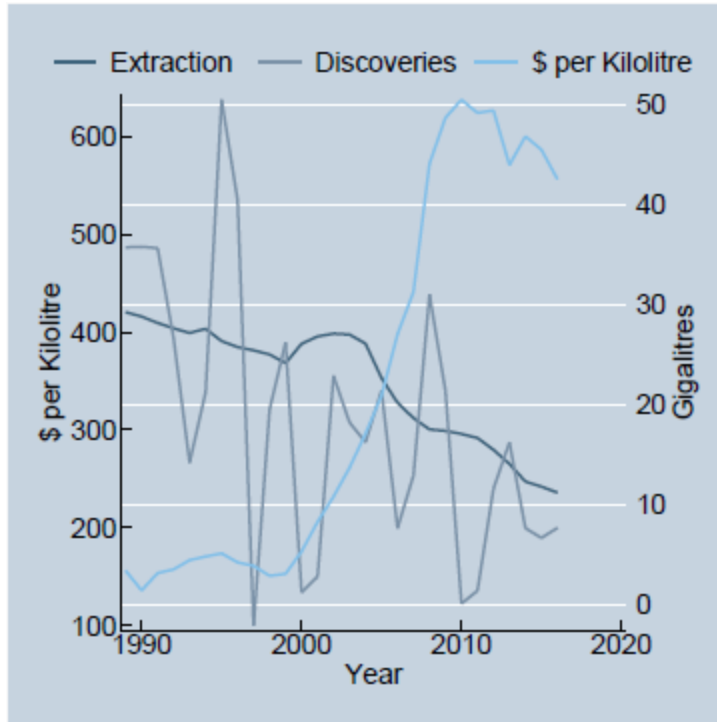
# Valuation of Natural Capital: Traditional User Cost, **Choices!**

Model	Natural Capital			Produced Capital
	$r_N$	$i_N$	$\pi_N$	$r_K$
$U_N^{T1}$	RBA Business Loan rate	Price deflater	Yes	RBA cash rate
$U_N^{T2}$	RBA Business Loan rate	Exponential smoothing	Yes	RBA cash rate
$U_N^{T3}$	RBA Business Loan rate	Geometric smoothing	Yes	RBA cash rate
$U_N^{T4}$	RBA Business Loan rate	Price deflater	No	RBA cash rate
$U_N^{T5}$	RBA cash rate	Price deflater	Yes	RBA cash rate
$U_N^{T6}$	RBA cash rate	Exponential smoothing	Yes	RBA cash rate
$U_N^{T7}$	RBA cash rate	Geometric smoothing	Yes	RBA cash rate
$U_N^{T8}$	RBA cash rate	Price deflater	No	RBA cash rate
$U_N^{T9}$	K Endogenous rate	Price deflater	Yes	RBA cash rate
$U_N^{T10}$	K Endogenous rate	Exponential smoothing	Yes	RBA cash rate
$U_N^{T11}$	K Endogenous rate	Geometric smoothing	Yes	RBA cash rate
$U_N^{T12}$	K Endogenous rate	Price deflater	No	RBA cash rate
$U_N^{T13}$	K and N Endogenous rate	Price deflater	Yes	K and N Endogenous rate
$U_N^{T14}$	K and N Endogenous rate	Exponential smoothing	Yes	K and N Endogenous rate
$U_N^{T15}$	K and N Endogenous rate	Geometric smoothing	Yes	K and N Endogenous rate
$U_N^{T16}$	K and N Endogenous rate	Price deflater	No	K and N Endogenous rate

# Iron Ore: Extraction, Discoveries, Price and User Cost Values

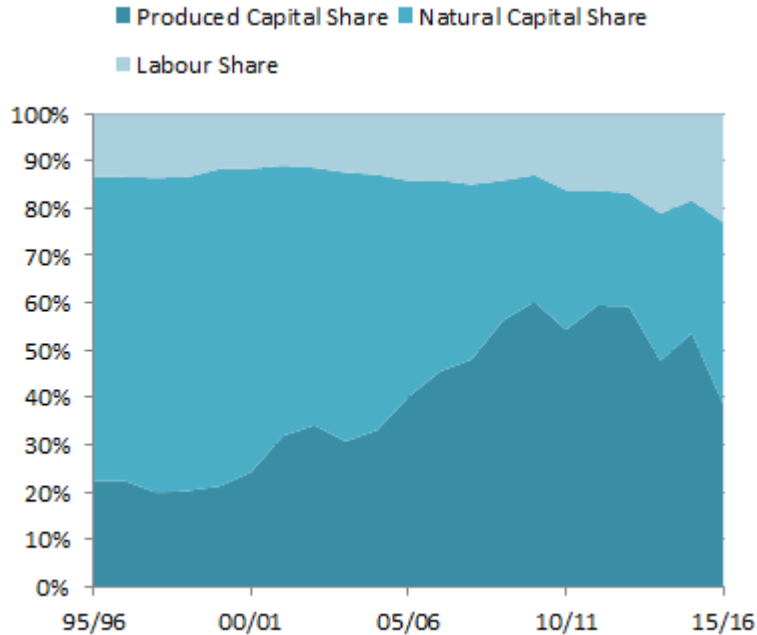


# Crude Oil: Extraction, Discoveries, Price and User Cost Values

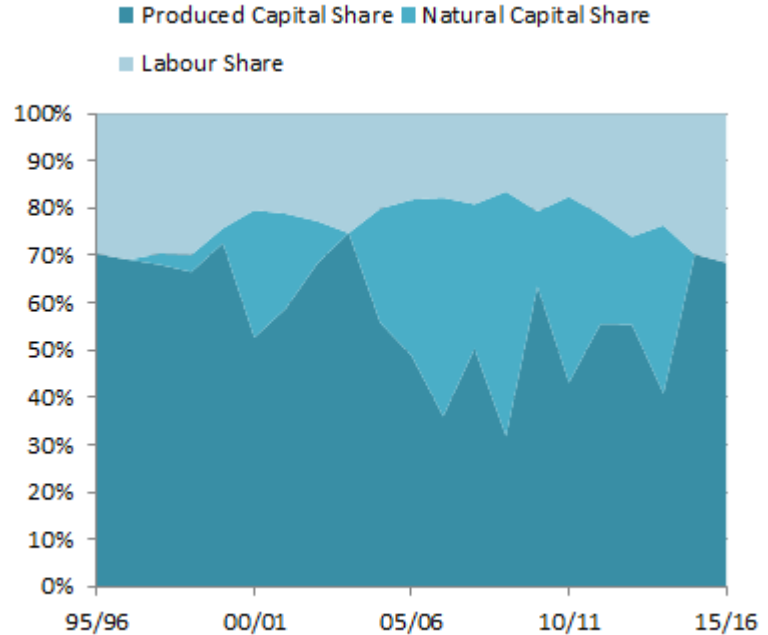


# Factor Cost Shares

## (a) Resource Rent



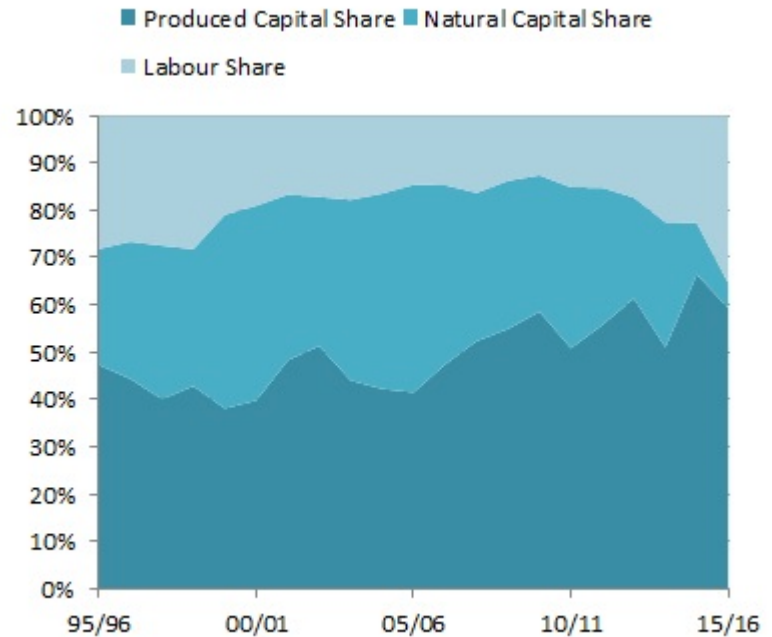
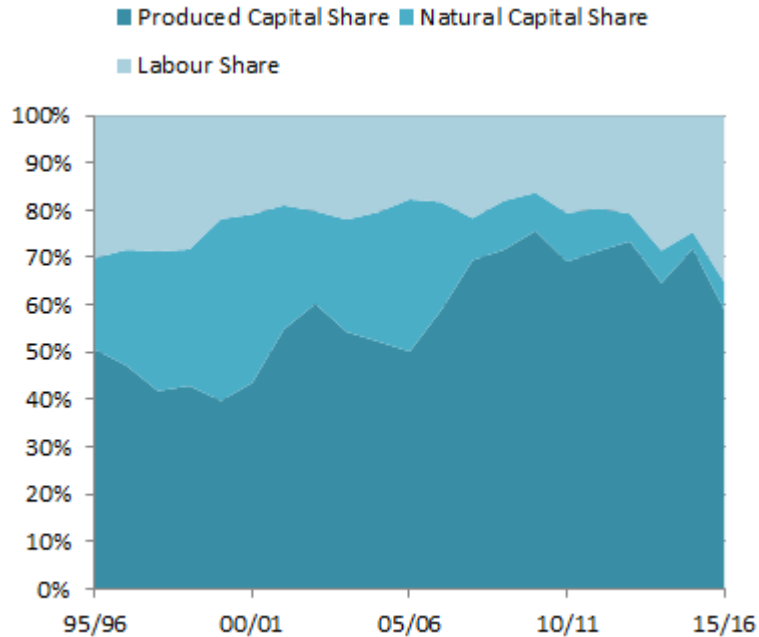
## (b) Residual Method



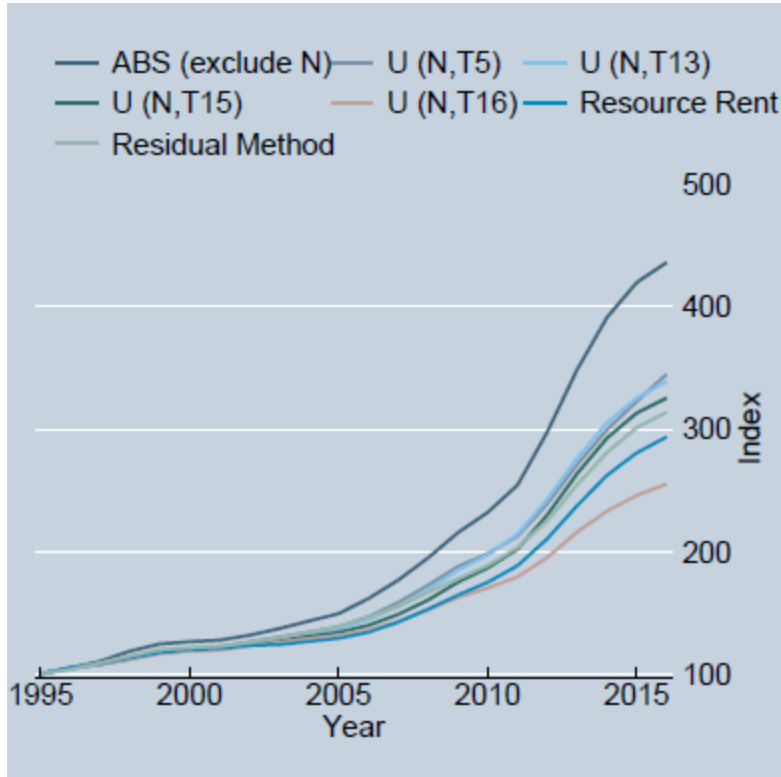
# Factor Cost Shares: User Cost

## (a) Diewert-Fox Method, U(15)

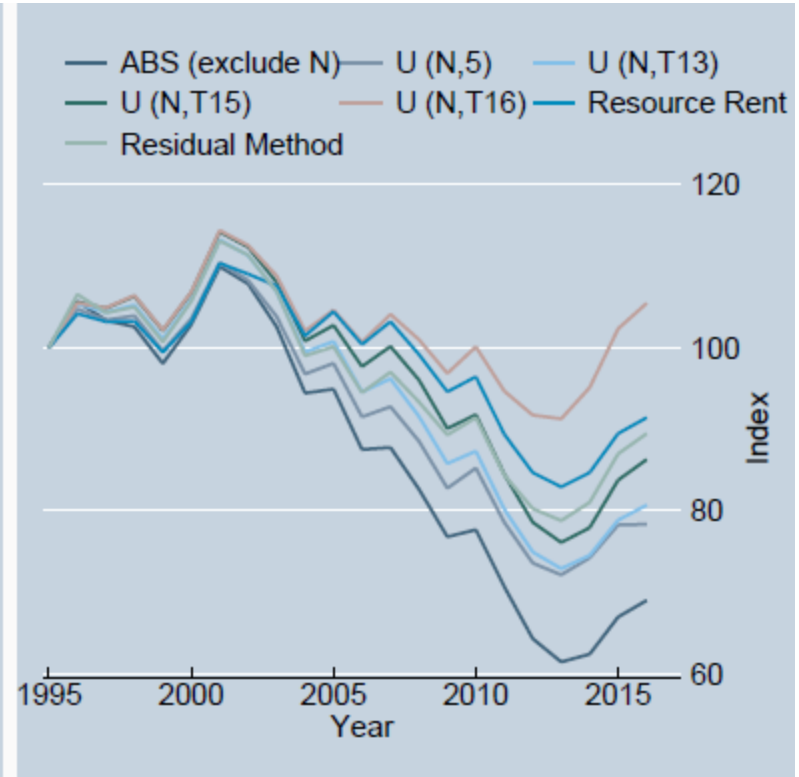
## (b) No Capital Gains, U(16)



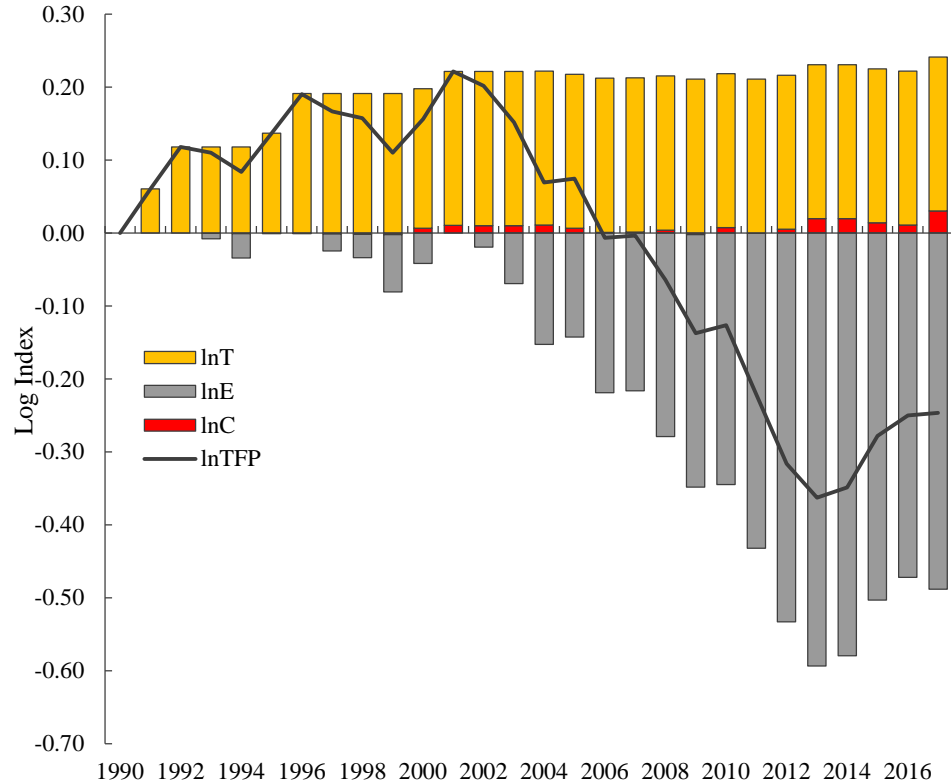
## (a) Capital Services



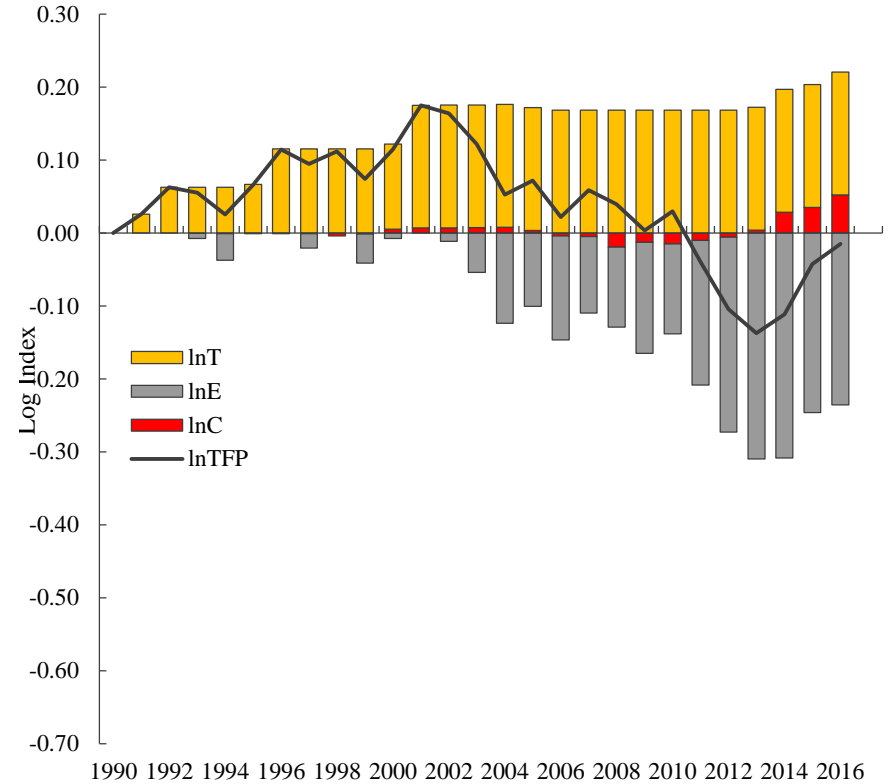
## (b) Multifactor Productivity



**(a) Without Natural Capital: ABS**

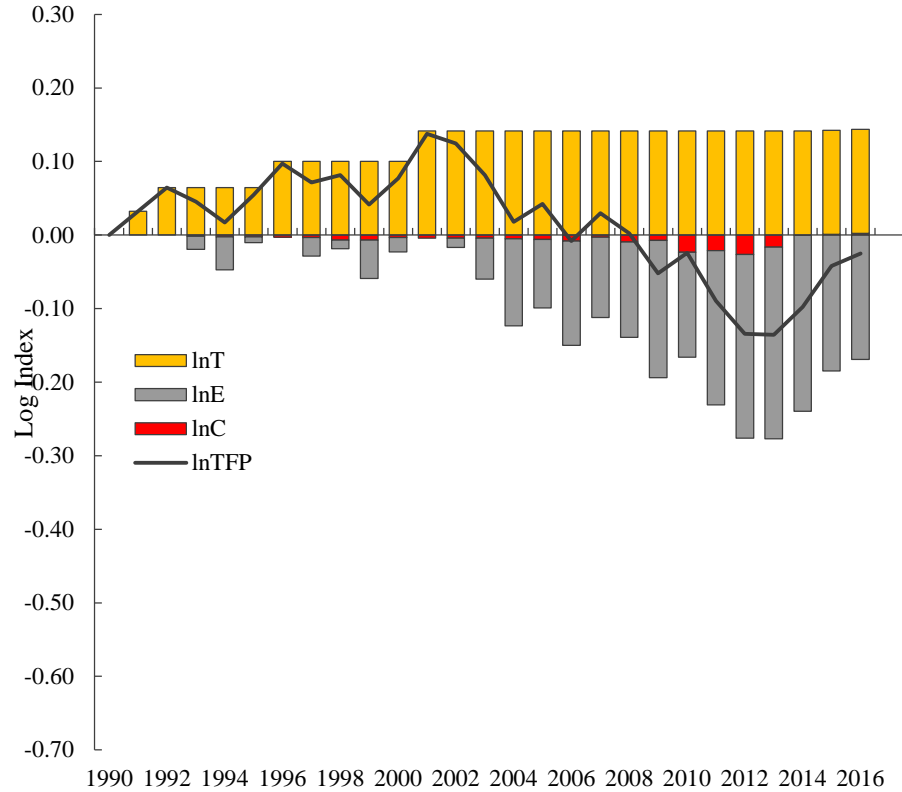


**(b) With Natural Capital: DF Method, U15**

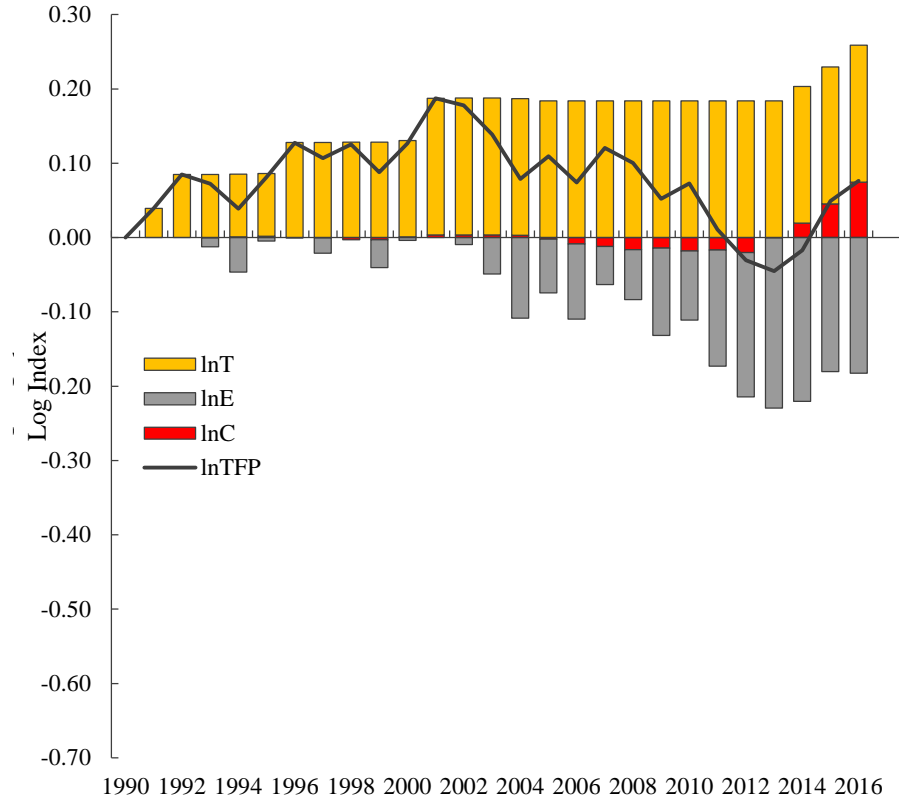




### (c) Resource Rent Method (Residual Method Almost Identical)



### (d) No Capital Gains: U16



# Conclusions

- Have used an innovative decomposition of value added.
- Looked at the impact on productivity and its sources from the introduction of natural capital.
- Considered alternative valuation methods for natural capital.
- Differing results can be found depending on the valuation method, but the key thing is to include natural capital.
- **The role of inefficiency highlighted, rather than negative TFP growth indicating technical regress.**
- Methods are easily implementable by NSOs.