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# ESCoE Research Seminar

GDP is a measure of output, not welfare.  
Or, HOS meets the SNA

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# GDP is a measure of output, not welfare. Or, HOS meets the SNA

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# GDP: welfare or output?

- Many people want to treat GDP as a measure of welfare. BUT:
- Many valid criticisms of GDP as a measure of welfare, e.g.
  - It is gross of depreciation (capital consumption)
  - It takes no account of inequality
  - It ignores things like health and life expectancy
  - The production boundary is arbitrary (unpaid housework and childcare excluded, imputed rent of owner-occupiers included)

# My view

- GDP can be an *indicator* of welfare, even though it is not a good *measure* of welfare.
- But we can do better: we can use the SNA to develop genuine measures of welfare.
- GDP is a measure of *output*. Output still matters a lot so we need to measure it well.

# THE ISSUE TODAY

This presentation has a narrower focus:

How should exogenous changes in the **terms of trade** (the export price relative to the import price) be treated? Do they change GDP? Or both welfare and GDP? Or just welfare?

To answer this, I use two textbook models of a small open economy. I ask

- How would an economic theorist answer these question?
- Would a national income accountant, with access to all the necessary data and applying SNA 2008, agree?

# Hecksher-Ohlin-Samuelson (HOS) model of a trading economy

- Small open economy. Terms of trade are exogenous.
- 2 goods. The country exports good 1.
- Technology and factor endowments (land and labour) are fixed. *No saving or investment.*
- Constant returns to scale and perfect competition, so prices equal marginal costs and factors paid their marginal products.

# Two models

## *Model 1*

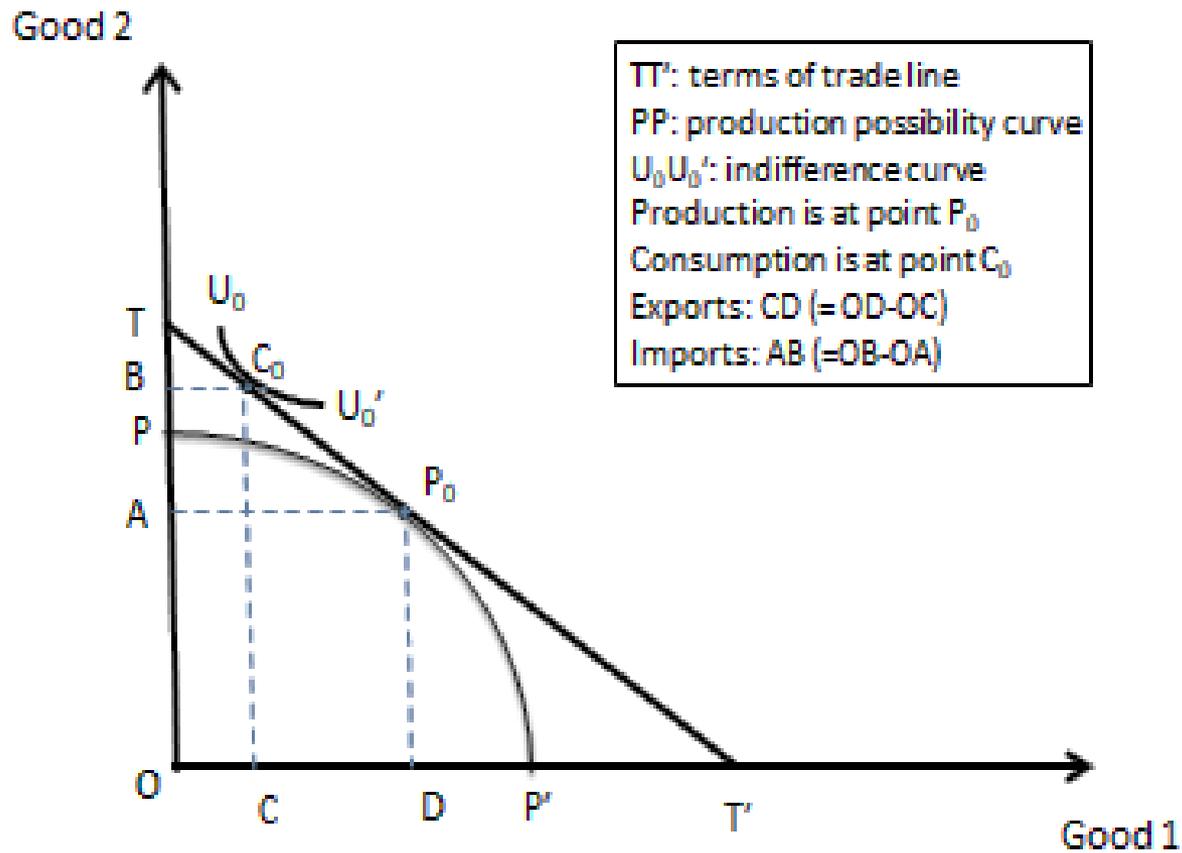
The country produces both goods which are for final consumption. Good 1 is exported, good 2 imported.

## *Model 2*

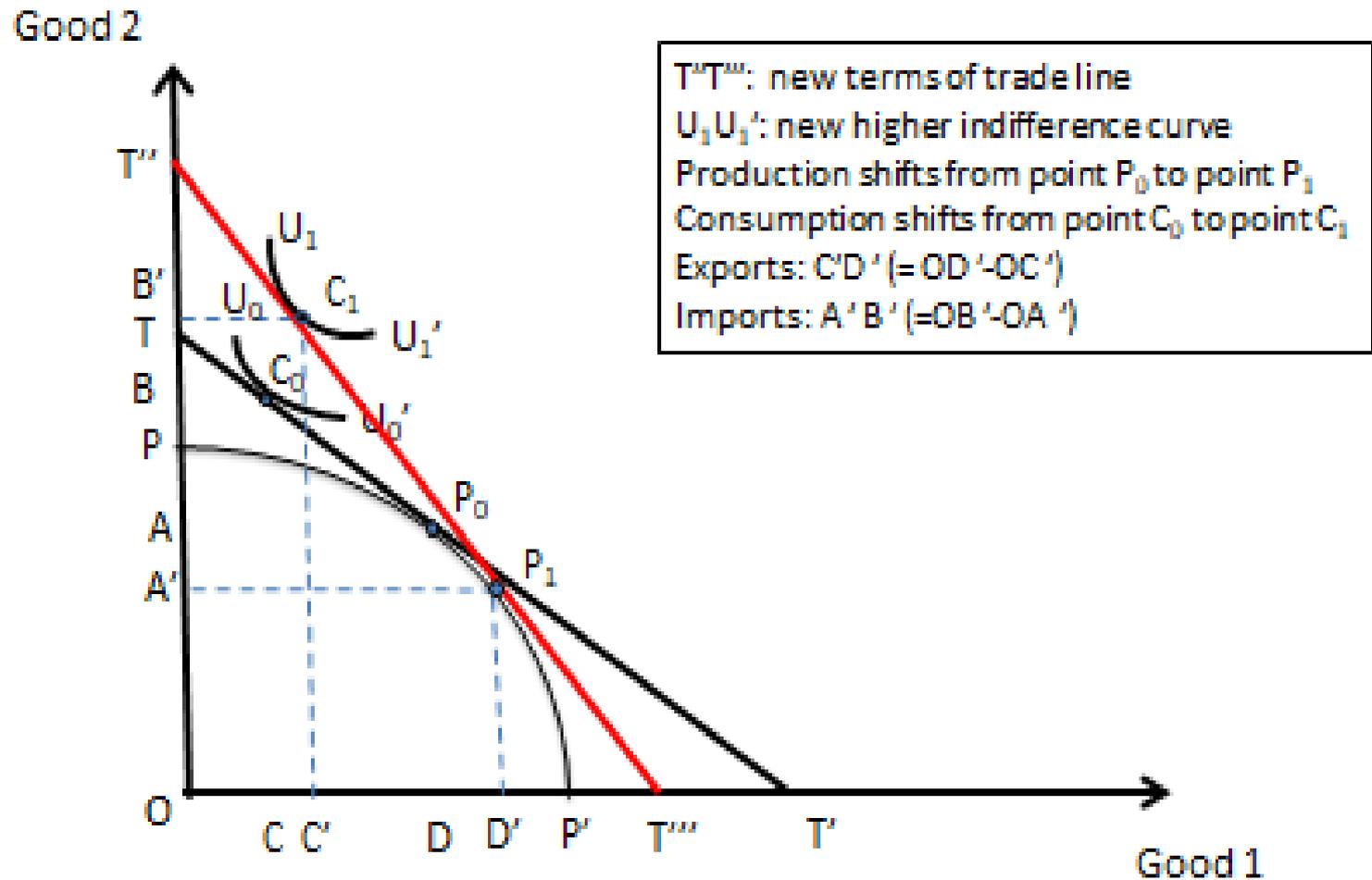
The country specialises on good 1. Good 1 is for final consumption and export, good 2 is an imported intermediate input (e.g. energy) into good 1.

(Bruno and Sachs 1985; Barsky and Kilian 2002; Blinder and Rudd 2008)

# Model 1



# Model 1, after improvement in terms of trade



# The theorist's answer for Model 1

- Welfare increases since the economy moves to a higher indifference curve.
- There has been a reallocation of resources towards the export good but aggregate output is unchanged.
- Reason: the economy moves *around* the production possibility curve but the curve does not shift since factor endowments and technology are fixed.

# Model 1: in maths

The production possibility frontier (or transformation curve):

$$F(Y_1, Y_2; R, L, \tau) = 0$$

$Y_1, Y_2$ : outputs of goods 1 and 2

$R, L$ : endowments of land ( $R$ ) and labour ( $L$ ), both constant

$\tau$ : level of technology (constant)

Differentiating the p.p. frontier with respect to time:

$$\frac{dY_2}{dt} + \left( \frac{\partial F / \partial Y_1}{\partial F / \partial Y_2} \right) \frac{dY_1}{dt} = 0$$

Consider  $\left( \frac{\partial F / \partial Y_1}{\partial F / \partial Y_2} \right)$ , the marginal rate of transformation between goods 1 and 2.

In a competitive economy

$$\left( \frac{\partial F / \partial Y_1}{\partial F / \partial Y_2} \right) = \frac{P_1}{P_2}$$

Hence after a bit of routine algebra

$$s_{Y_1}^{GDP} \hat{Y}_1 + s_{Y_2}^{GDP} \hat{Y}_2 = 0 \quad [\text{A hat denotes a growth rate, e.g. } \hat{Y}_1 = d \ln Y_1 / dt]$$

$s_{Y_1}^{GDP}, s_{Y_2}^{GDP}$  : shares of goods 1 and 2 in the total value of output (GDP)

# Divisia indices

For Model 1 we have just found that

$$s_{Y_1}^{GDP} \hat{Y}_1 + s_{Y_2}^{GDP} \hat{Y}_2 = 0$$

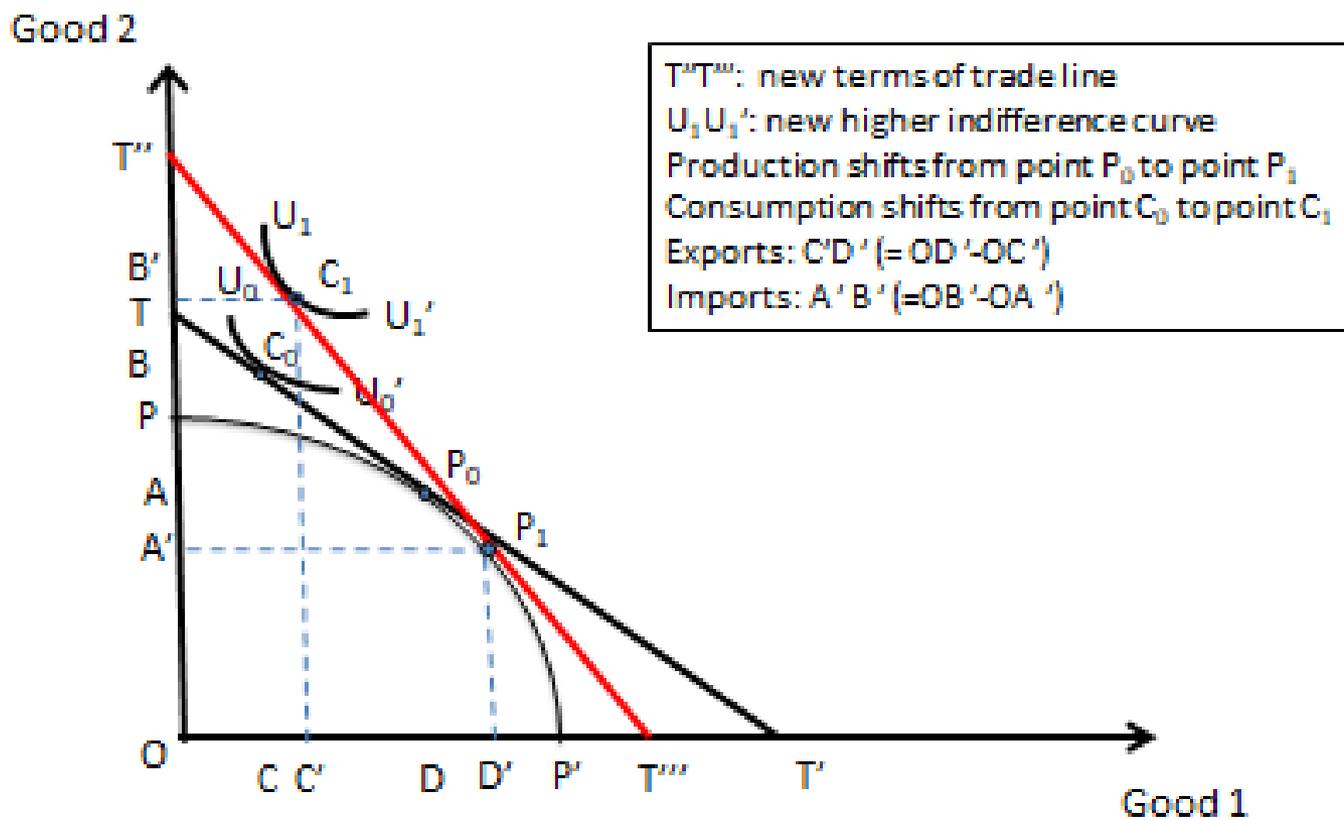
The left hand side is a Divisia index of aggregate output  $Y$  :

$$\hat{Y} = s_{Y_1}^{GDP} \hat{Y}_1 + s_{Y_2}^{GDP} \hat{Y}_2$$

So we have shown that under the assumptions of Model 1

$$\hat{Y} = 0$$

*Improvement in terms of trade is thought of as taking place over a finite interval, say (0,T)*



# Total change over a finite interval

The Divisia index of aggregate output gives the instantaneous growth rate at a point in time. The total proportionate change over a finite interval  $(0, T)$  is given by integration:

$$\log \left( \frac{Y(T)}{Y(0)} \right) = \int_0^T \hat{Y}(t) dt$$

= 0 in this case.

[Memo: this is the continuous analogue of a discrete chain index.]

# Divisia indices

Suppose we have some aggregate

$$Z = \sum_{i=1}^{i=N} p_i q_i$$

We want to split this into an average price  $P$  and an average quantity  $Q$ :

$$PQ = \sum_{i=1}^{i=N} p_i q_i = Z$$

Calculus tells us that

$$\hat{P} + \hat{Q} = \sum_{i=1}^{i=N} s_i \hat{p}_i + \sum_{i=1}^{i=N} s_i \hat{q}_i \quad s_i := p_i q_i / Z, \text{ the shares}$$

So define

$$\hat{P} := \sum_{i=1}^{i=N} s_i \hat{p}_i \quad \hat{Q} := \sum_{i=1}^{i=N} s_i \hat{q}_i$$

NB: the shares can change continuously with time.

# Properties of Divisia indices

1. Price index times quantity index equals value index:  $PQ = Z$
2. Divisia indices are consistent in aggregation

These properties are not shared by any index number in common use.

Only one problem: Divisia indices are defined in continuous time!

# François Divisia (1889-1964)



# National accounts in Model 1

Supply Use relationships:

$$P_1 Y_1 = P_1 C_1 + P_1 X_1$$

$$P_2 Y_2 + P_2 M_2 = P_2 C_2$$

GDP:

$$GDP(E) := P_E E \equiv P_1 C_1 + P_2 C_2 + P_1 X_1 - P_2 M_2$$

$$GDP(O) := P_Y Y \equiv P_1 Y_1 + P_2 Y_2$$

Easy to see that

$$GDP(E) = GDP(O)$$

$$\hat{P}_E = \hat{P}_Y$$

$$\hat{E} = \hat{Y}$$

## Would the national income accountant agree with the theorist about Model 1? (1)

Yes.

If the national income accountant measured growth in real GDP by a Divisia index, he/she would find that *empirically*

$$\hat{Y}(t) = 0, \quad 0 \leq t \leq T$$

So the total change in GDP between 0 and  $T$  is zero, in agreement with the theorist.

# A national-accounts-based welfare measure

- Real consumption is a good measure of welfare in Model 1.
- If household demand is homothetic, a Divisia price index of consumption is also a true cost-of living index.
- So welfare can be measured *either* by nominal consumption deflated by the Divisia *price* index of consumption or by the Divisia *quantity* index of consumption.

# Welfare versus output

- In Model 1, nominal consumption is equal to nominal GDP because the trade balance is zero.
- But real consumption is growing faster than real GDP, when both are measured by Divisia indices:

$$\hat{C} = s_M^{GDP} (\hat{P}_1 - \hat{P}_2) = s_M^{GDP} \hat{p} > 0$$

$p$  : terms of trade

$s_M^{GDP}$  : share of imports in GDP

## **Would the national income accountant agree with the theorist about Model 1? (2)**

- Yes.
- The national income accountant would find that real consumption had increased while real GDP had stayed constant.
- It follows that GDP is a measure of output but not a measure of welfare.
- But the SNA does allow one to measure welfare using real consumption.

# National accounts in Model 2

Supply Use relationships:

$$P_1 Y_1 = P_1 C_1 + P_1 X_1$$

GDP:

$$GDP(E) := P_E E \equiv P_1 C_1 + P_1 X_1 - P_2 M_2$$

$$GDP(O) := P_Y Y \equiv P_1 Y_1 - P_2 M_2$$

Easy to see that:

$$GDP(E) = GDP(O)$$

$$\hat{P}_E = \hat{P}_Y, \quad \hat{E} = \hat{Y}$$

Divisia index for GDP:

$$\hat{Y} = (1 + s_M^{GDP}) \hat{Y}_1 - s_M^{GDP} \hat{M}_2$$

## Model 2: theory

Production function for good 1:

$$Y_1 = Y_1(R, L, M_2, \tau)$$

Land ( $R$ ), labour ( $L$ ) and technology ( $\tau$ ) assumed fixed. So fall in relative price of imported energy ( $M$ ) leads to increase in volume of imports which raises output of good 1:

$$\dot{Y}_1 = \frac{\partial Y_1}{\partial M_2} \dot{M}_2$$

or

$$\hat{Y}_1 = \left( \frac{s_M^{GDP}}{1 + s_M^{GDP}} \right) \hat{M}_2 > 0$$

assuming inputs are paid their marginal products.

## Back to the national accounts ...

**Empirically**, the national income accountant in model 2 would then find:

$$\hat{Y} = (1 + s_M^{GDP}) \hat{Y}_1 - s_M^{GDP} \hat{M}_2 = 0$$

i.e. *no change in GDP*. But welfare (real consumption of good 1) *rises*:

$$\hat{C}_1 = -s_M^{GDP} (\hat{X}_1 - \hat{M}_2)$$

$$= s_M^{GDP} \hat{p} > 0 \quad p : \text{terms of trade}$$

NB: volume of imports rises faster than that of exports (trade balance is zero).

# Summary

- In both models, GDP is **constant** but welfare (real consumption) **rises** when the terms of trade improve.
- This is because the models are static. In Model 2 allowing capital to be accumulated would cause GDP to increase.
- There is a third case: the second good is an imported *capital* good. Improving terms of trade now lead to faster capital accumulation so that both GDP and consumption (welfare) rise (Oulton 2012).

## Coming down to earth a bit ...

- Data are discrete, not continuous. So Divisia indices can't be calculated.
- What do SNA 2008 and ESA 2010 say about changes in the terms of trade?

## Real world indices

- Annually-chained indices (Laspeyres, Fisher or Törnqvist) are now the standard. They are the real world counterparts of Divisia indices
- They can be regarded as approximations to Divisia indices.
- Perhaps the approximations may get better if the world moves over time to quarterly or even monthly chaining? (It's already moved from fixed base to annual chaining).

# What does SNA 2008 say?

- Chapter 15: “Real GDI = Real GDP + Trading gain”. Real GDI is a welfare measure, a step on the road to “Real net national disposable income”.
- Trading gain is:

$$\text{Trading gain} = \frac{P_X X - P_M M}{P} - (X - M)$$

- In Models 1 and 2, the first term is zero. The second term corresponds to the effect on real consumption derived for Models 1 and 2.
- So no conflict here.

# The trade balance

- Both Models 1 and 2 exclude saving and investment and therefore assume a zero trade balance.
- In a closed economy, the best welfare measure is net national income deflated by a consumption price index. I.e. net investment should be deflated by the consumption price index, not by the investment price index. (Weitzman 1976; Sefton and Weale 2006).
- In an open economy, Sefton and Weale (2006) argue that the trade balance should also be deflated by the consumption price index.
- SNA 2008 is agnostic on this. But *unfortunately* ESA 2010 recommends deflating by an average of export and import price indices.

# Conclusions

- GDP is a measure of output, not welfare.
- But welfare can be measured within the framework of the SNA.
- Divisia indices provide a sound conceptual basis for measuring prices and volumes in the national accounts.

**THE END**